Assessing Uncertainty in Fertility Estimates and Projections

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Abstract

The United Nations Population Division produces estimates and projections of the total fertility rate for all countries in the world every two years. For countries with fertility above replacement level, future levels are projected based on a parametric function. We develop a Bayesian hierarchical model for producing probabilistic projections of fertility. Differences in data quality for observations within a country are assessed by relating a standardized weighting scheme to the empirical variance of the measurement errors. We quantify the country-specific uncertainty in future fertility, as well as in the estimates of past levels of fertility, and we give results for a number of Asian and African countries.

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1 INTRODUCTION

The UN Population Division produces estimates and projections of various demographic indicators for all countries in the world every two years. The latest revision was produced in 2007 and is the 2006 Revision of the World Population Prospects (United Nations, Department of Economic and Social Affairs, Population Division 2007). A demographic transition model is used to project the decline in the total fertility rate (TFR) from high levels of fertility towards replacement level fertility. In this model the annual decrements in the TFR are a function of the level of fertility (United Nations, Department of Economic and Social Affairs, Population Division 2005). Three sets of parameter values describe three different trajectories of future declines, from which the analyst chooses one which seems most appropriate for the country of interest. This gives the *Medium variant* of the fertility projection. The effect of lower or higher fertility is illustrated with the *Low* and *High* variants of the projections. In the high variant, half a child is added to the medium variant in order to examine the influence of a slower fertility decline on the population projections. Similarly for the low variant, half a child is subtracted from the medium variant.

A drawback with this approach is that the projections are not very country-specific; only three trajectories are being considered for modeling future fertility decline from which one is chosen. This means that the medium variant gives the same pace of fertility decline for several countries, regardless of the difference in pace of the decline that has been observed in the past. Moreover, this approach does not assess the uncertainty in future fertility levels between countries (Bongaarts and Bulatao 2000) and it does not give insight into the difference in uncertainty between countries; Countries in which the fertility transition has only just started will have more uncertainty in future levels of fertility than countries for which fertility is close to replacement level. Last, measurement errors in fertility rates lead to uncertainty in past estimates of fertility rates. The extent of the uncertainty in past fertility levels is not being assessed either.

In this article we propose a model for (a) deriving country-specific projections of the total fertility rate and (b) assessing the uncertainty around estimates and projections of the total fertility rate during the fertility transition from high fertility towards replacement fertility. We propose a Bayesian hierarchical model to produce probabilistic estimates and projections. In this model, fertility decline is decomposed into a systematic decline with distortion terms added to it. The pace of the systematic decline in TFR is modeled as a function of its level, based on the UN methodology. Difference in data quality is taken into account by relating a standardized weighting scheme to the variance of the measurement errors.

This article is organized as follows; In Section 2 we discuss the model as used by the UN Population Division to predict fertility decline, Section 3 explains the Bayesian hierarchical model which is used to estimate and predict fertility and asses the uncertainty in TFR over time. Section 4 deals with data issues, how to incorporate the differences in data quality into the model. In Section 5 we present results for a number of countries and in Section 6 we discuss possible improvements on the methodology.

2 MODELING FERTILITY DECLINE

The UN Population Division uses a demographic transition model to project fertility decline for countries in which the TFR is above replacement level. Annual fertility decrements are modeled as a function of the level of fertility (United Nations, Department of Economic and Social Affairs, Population Division 2005; Meyer 1994). Fertility decline starts slowly at high TFR values, the pace increases and peaks around a total fertility rate of 5 children per woman, and then slows down again towards the end of the transition.

The UN uses three different trajectories to model future declines. The three trajectories are based on observed fertility declines in countries that have completed the fertility transition. The "Slow/Slow" trajectory represents a slow-paced fertility decline, the "Fast/Fast" trajectory represents a decline with a faster pace. The third trajectory is called "Fast/Slow", which represents a decline that starts at a high pace like the Fast/Fast scenario, then over time its pace matches up with the Slow/Slow trajectory. For each country, the analyst chooses one trajectory which seems most appropriate for the future fertility decline in that country. Figure 1 shows the Slow/Slow and Fast/Fast trajectories, together with the annual decrements since 1950 for all countries going through the fertility transition. These annual decrements are derived from the estimates of past fertility levels in the UN World Population Prospects (WPP), the 2004 revision (United Nations, Department of Economic and Social Affairs, Population Division 2005; United Nations, Department of Economic and Social Affairs, Population Division 2005).

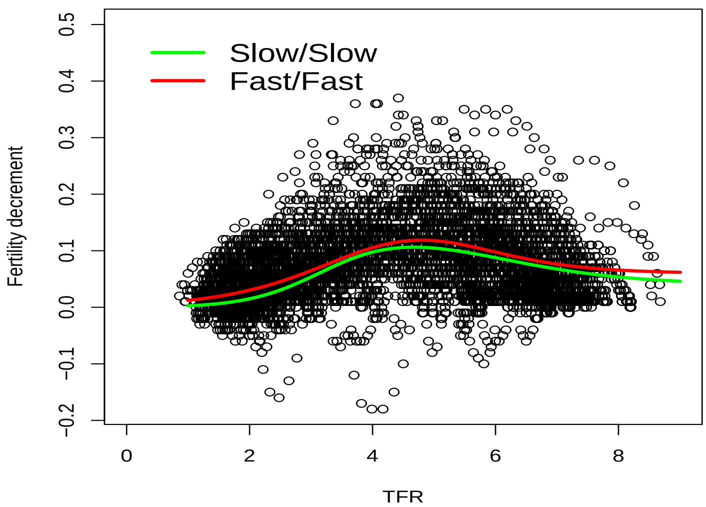


Figure 1: Annual decrements in TFR: The Slow/Slow and Fast/Fast scenarios for predicting fertility decline (in green and red), and observed annual fertility decrements since 1950 as given by the UN WPP 2004 estimates.

The pace of the fertility decline is modeled as a function of its level using the sum of two logistic curves, a double logistic function (Meyer 1994). The first logistic curve describes a high pace of decline at high total fertility rates decreasing towards a lower pace for smaller fertility. The second curve describes the opposite effect; an increase in TFR for higher values to slow down the pace of fertility decline at the beginning of the transition. The sum of the two describes a decline in fertility which is low for high and low TFR, and peaks around 5. The scenarios are based on different sets of parameters of the double logistic function.

The model for the pace of the fertility decline is denoted as follows; Let f be the total fertility rate, and $d(f)$ the the annual decrement (fertility decline) at TFR f as modeled by the double logistic function, which is given by:

$$d(f) = d_{max} \left(\frac{-1}{1 + \exp\left(-\frac{ln(p^2)}{\triangle_1}(f - t_1)\right)} + \frac{1}{1 + \exp\left(-\frac{ln(p^2)}{\triangle_3}(f - t_3)\right)} \right), \qquad (1)$$

with parameters $(\triangle_1, \triangle_2, \triangle_3, \triangle_4, d_{max})$, $t_1 = \triangle_4 + \triangle_3 + \triangle_2 + 0.5\triangle_1$ and $t_3 = \triangle_4 + 0.5\triangle_3$. For an interpretation of the parameters $(\triangle_1, \triangle_2, \triangle_3, \triangle_4, d_{max})$, see Table 1 and Figure 2.

Note that the double logistic model does not predict the onset of the fertility transition, it focuses on pace and trajectory of the decline after its onset. In order to predict future fertility levels in countries for which a decline has not yet been observed, additional assumptions are needed about the timing of the onset of the decline.

Parameter	**Interpretation**
d_{max}	Maximum annual decrement
$\triangle_1$	TFR range for which decrements increase from $\frac{1}{p+1}d_{max}$ to $\frac{p}{p+1}d_{max}$
$\triangle_2$	TFR range with approximately constant decrements d_{max} (decrements change from $\frac{p}{p+1}d_{max}$ to d_{max} to $\frac{p}{p+1}d_{max}$)
$\triangle_3$	TFR range for which decrements decrease from $\frac{p}{p+1}d_{max}$ to $\frac{1}{p+1}d_{max}$
$\triangle_4$	TFR range for which decrements decrease from $\frac{1}{p+1}d_{max}$ to 0
For $p = 9$	
$\triangle_1 + \triangle_2 + \triangle_3 + \triangle_4$	TFR level at the onset of fertility decline at which the pace is $0.1d_{max}$
$\triangle_2 + \triangle_3 + \triangle_4$	TFR at which the pace has increased to $0.9d_{max}$
$\triangle_3 + \triangle_4$	TFR at which the pace is $0.9d_{max}$ and has started to decrease
$\triangle_4$	TFR at which the fertility levels off (pace is $0.1d_{max}$ and will decrease to zero)

Table 1: Parameters of the double logistic function

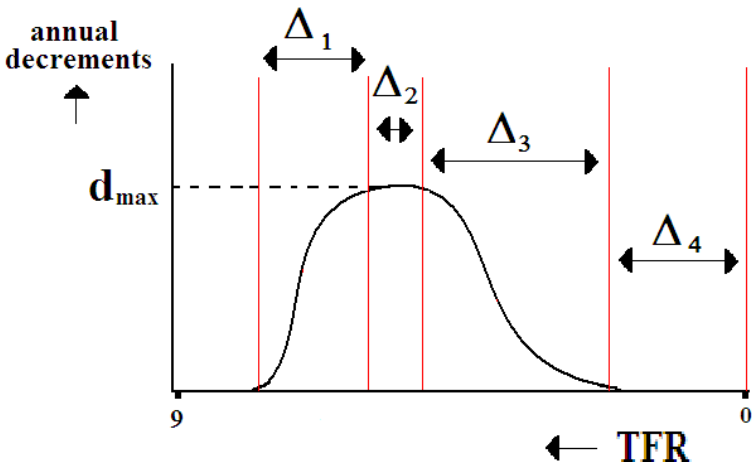


Figure 2: The double logistic model for fertility decrements; Annual decrements are plotted versus decreasing TFR.

3 BAYESIAN HIERARCHICAL MODEL

In this section we discuss a Bayesian hierarchical model for fertility decline. The goals of this model are to (a) construct country-specific projections of fertility decline, (b) assess the uncertainty around estimates and projections of the total fertility rate during the fertility transition from high fertility towards replacement fertility.

Denote the TFR for country c, year t by f_{ct}. The length of the observation period is T, $t \in \{1, \ldots, T\}$, there are C countries, $c \in \{1, \ldots, C\}$. Now let y_{cts} be observed TFR for country c in year t for observation $s = 1, \ldots, n_t$ with:

$$y_{cts} \overset{\text{ind}}{\sim} N(f_{ct}, \sigma^2_{cts}), \tag{2}$$

with σ^2_{cts} being the variance of the measurement error of observation y_{cts}. The various observations of TFRs differ in quality, some observations are known to be of better quality than other observations; the measurement errors differ between observations. The variance of the measurement errors will be discussed in the next section. It will be modeled as

$$\sigma_{cts} = M_{cts}\delta_c \tag{3}$$

with M_{cts} a "data quality multiplier" which is determined by data quality covariates of observation y_{cts}, and δ_c the country-specific standard deviation of the measurement errors.

Data quantity differs substantively between countries. For example, in Bangladesh we have time series with several observations of the TFR in each year since 1970. For Bhutan, there are only 5 observations in total of TFR levels in the past. The model for fertility decline should be flexible enough for countries with enough data to let the data determine what the TFR looks like. However, as the UN produces estimates and projections for all countries, the modeling assumptions need to be strong enough for the results to be useful/realistic for countries with less data. Our approach to deal with these two different situations is to decompose fertility decline into a smooth systematic decline part, with "distortions' added to it. The systematic decline represents the fertility transition from high fertility to replacement fertility (or something close to it) as a smooth time series. The systematic decline is modeled with the double-logistic function as described in the previous section. In reality, the TFR does not need to be a smooth curve, which is taken into account by adding the distortions to the smooth decline curve. For years without data, the most appropriate trajectory for the TFR will be given by the smooth decline curve with a random distortion term centered around zero added to it. For years with data, the distortion will be more likely to be positive (negative), if the observations are higher (lower) than the trend as given by the smooth curve. An example is shown is Figure 3 for Burkina Faso.

Let $\tilde{f}_{ct}$ represent the TFR as modeled by the double logistic function. A distortion term for country c in year t, denoted by ε_{ct}, is added to get the "true" TFR, f_{ct}:

$$f_{ct} = \tilde{f}_{ct} + \varepsilon_{ct}, \tag{4}$$

In general, we expect the TFR to follow the theoretical trend, as modeled by assigning normal distribution to the distortion terms with mean zero, and the distortion in one year to be independent of the distortions in another year:

$$\varepsilon_{ct} \overset{\text{ind}}{\sim} N(0, \sigma^2_{\varepsilon}). \tag{5}$$

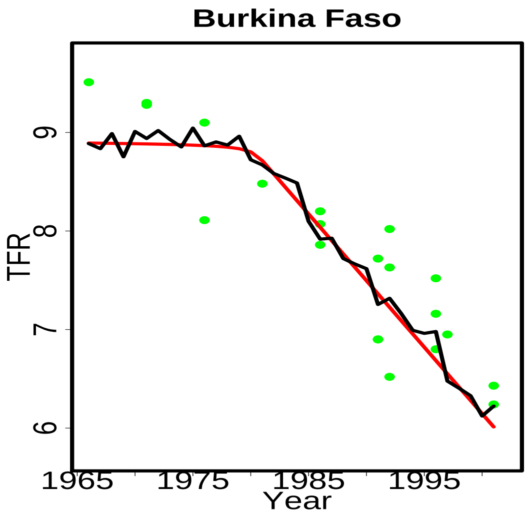


Figure 3: TFR in Burkina Faso; Observations from multiple sources (green), a trajectory of the TFR (black), and the TFR as modeled by the double logistic function (red).

The variance of the distortions, σ_ε^2, is the same for all countries, reflecting the assumption that the extent to which the real TFR can divert from the systematic decline is comparable among countries.

With the same notation as before, the annual decrements $d_c(\tilde{f}_{ct})$ as given by the double logistic function at TFR level $\tilde{f}_{ct}$ are given by:

$$d_c(\tilde{f}_{ct}) = d_{cmax}\left(\frac{-1}{1+\exp\left(-\frac{ln(p^2)}{\triangle_{c1}}(\tilde{f}_{ct}-t_{c1})\right)}+\frac{1}{1+\exp\left(-\frac{ln(p^2)}{\triangle_{c3}}(\tilde{f}_{ct}-t_{c3})\right)}\right), \tag{6}$$

with parameters $\boldsymbol{\theta}_c = (\triangle_{c1}, \triangle_{c2}, \triangle_{c3}, \triangle_{c4}, d_{cmax})$, $t_{c1} = \triangle_{c4} + \triangle_{c3} + \triangle_{c2} + 0.5\triangle_{c1}$, $t_{c3} = \triangle_{c4} + 0.5\triangle_{c3}$ and p is set to 9. The level of $\tilde{f}_{ct}$ is determined by the parameters of the double logistic function combined with a given level of the TFR in a particular year. Define τ_c as the start year of the fertility transition, at which the the pace of the decline is $0.1d_{c,max}$. Then $\tilde{f}_{ct}$ is given by:

$$\tilde{f}_{ct} \quad \approx \quad \begin{cases} \tilde{f}_{c,t-1} + d_c(\tilde{f}_{c,t-1}), & \text{for } t < \tau_c. \\ \sum_i \triangle_{ci}, & \text{for } t = \tau_c \\ \tilde{f}_{c,t-1} - d_c(\tilde{f}_{c,t-1}), & \text{for } t > \tau_c. \end{cases} \tag{7}$$

Note that we use an “approximately equal to” sign in (7) because the TFR in year t before the start year τ_c is not given by $\tilde{f}_{c,t-1} + d_c(\tilde{f}_{c,t-1})$, but instead by $\tilde{f}_{c,t-1} + d_c(\tilde{f}_{c,t})$; the pace of the decline is based on the TFR in the current year. However, as the pace of the decline is small before the start year (less than $0.1d_{c,max}$), the difference will be very small and the approximation as used in (7) avoids numerically difficulties when calculating the levels of fertility before the start year. The start year of the fertility decline for the countries considered here is assumed to be between 1950 and 2010:

$$\tau_c \quad \sim \quad \text{UniDiscrete}[1950, 2010]. \tag{8}$$

We take into account the uncertainty in the parameters of the double logistic function and assume that the parameter vectors are comparable among countries. A Bayesian hierarchical model is used to derive the country-specific distributions of each of the parameters (Gelman et al. 2004). As before, the parameter vector of the double logistic function for country c is given by $\boldsymbol{\theta}_c$ with $\theta_{ci} = \triangle_{ci}$ for $i = 1, \ldots, 4$ and $\theta_{c5} = d_{cmax}$. Denote the vector of the hierarchical mean parameters by $\boldsymbol{\alpha}$ such that

$$\theta_{ci} \sim N_{V_i}(\alpha_i, \sigma_i^2), \quad (9)$$

where α_i and σ_i^2 are the hierarchical mean and variance for parameter θ_i. N_{V_i} denotes a truncated normal distribution with outcomes in the interval V_i. The truncated normal is used to put restrictions on the parameters to get realistic decline curves, the intervals are given by $V_i = [0, 6]$ for $i = 1, 2, 3$, $V_4 = [1, 3]$ and $V5 = [0, 0.4]$ such that the the TFR ranges $\triangle_1$ to $\triangle_3$ are restricted to be between 0 and 6 children, the "asymptotic TFR" $\triangle_4$ is between 1 and 3 children and the maximum fertility annual decrement d_{max} is restricted to be between 0 and 0.4 child decrease per year.

The prior distributions for the hierarchical mean parameters are given by:

$$\alpha_i | \sigma_i^2 \sim N_{V_i}(\alpha_{i0}, \sigma_i^2 / \kappa_i), \quad (10)$$

with $\kappa_i = 1$. Conjugate prior distributions are used for the baseline country-specific error variance δ_c^2, the variance of the distortion terms σ_ε^2 and the variance of the parameters in the double logistic function σ_i^2 for $i = 1, \ldots, 5$:

$$\begin{aligned} \delta_c^2 &\sim \text{InverseGamma}(a_{y0}, b_{y0}), & (11)\\ \sigma_\varepsilon^2 &\sim \text{InverseGamma}(a_{\varepsilon 0}, b_{\varepsilon 0}), & (12)\\ \sigma_i^2 &\sim \text{InverseGamma}(a_i, b_i), & (13) \end{aligned}$$

with prior parameters $a.$ and $b.$ based on fertility decrements for countries that have completed the fertility transition as estimated in the UN WPP 2004.

The parameters in the model are given by $\{\alpha_i, \sigma_i^2, \theta_{ci}, \tau_c, \delta_c^2, \varepsilon_{ct}, \sigma_\varepsilon^2\}$. We use a Markov chain Monte Carlo sampling procedure to get samples of the posterior distributions of each of the parameters (Gelfand and Smith 1990). The posterior samples of $\{\theta_{ci}, \tau_c, \varepsilon_{ct}\}$ combined give the posterior sample of the TFR f_{ct}. The MCMC sampling algorithm is implemented in the statistical package R.

4 DATA QUALITY

The various observations of TFRs differ in quality, depending on a number of "data quality covariates", eg. the source (census, survey, vital registration, etc.), the time span on which the observation is based or estimation method. In this section we will discuss the set-up and use of a weighting scheme, based on the approach as used by UNICEF (Hill et al. 1998). Each observation y_{cts} is assigned a weight w_{cts}, that is determined by a number of data quality covariates. Larger weights are assigned to observations with data quality covariates that tend to give better quality information, eg. based on more recent periods or longer time spans.

A simplified version of the weight w_{cts} for observation y_{cts} is given by:

$$w_{cts} = \text{Source}(y_{cts}) * \text{Recall weight}(y_{cts}) * (1 + \log(\text{Time span}_{cts})), \tag{14}$$

with default illustrative weights for type of data source for developing countries given in Table 2. These default values should be thought of as a way to rank the different sources relatively to one another, they are initial guesses subject to revision on a case by case basis for each country and time period. The weights for time span and the recall weight (based on midpoint of period before survey) shown in Figure 4. The decay function for the recall weight in Figure 4(a) attempts to model the recall lapse problem with retrospective data, which is a well known and documented issue with census and survey questions collecting information for past events like lifetime fertility (i.e, total number of ever born children) or maternity histories - especially from older women (Som 1973; Potter 1977; Becker and Mahmud 1984; Pullum and Stokes 1997).

SOURCE	WEIGHT
Census	0.25
DSS, Prospective studies, Longitudinal panel	1
Sample (Vital) Registration System	1
Life Table	1
Population register	0.5
Vital Statistics/Registration	0.25
Demographic and Health Survey	1
Reproductive Health Survey	1
World Fertility Survey	0.9
Multiple Indicator Cluster Survey (UNICEF)	0.8
Cross-sectional survey	0.75
Undefined survey	0.75

Table 2: Weights for different types of data sources in developing countries.

Observations of worse quality are more likely to have larger measurement errors. As defined earlier, σ^2_{cts} is the empirical variance of the measurement errors for observation y_{cts}. The goal of examining data quality is to find a "data quality multiplier" M_{cts} for the empirical standard deviation of the measurement errors which reflects the quality of observation y_{cts}.

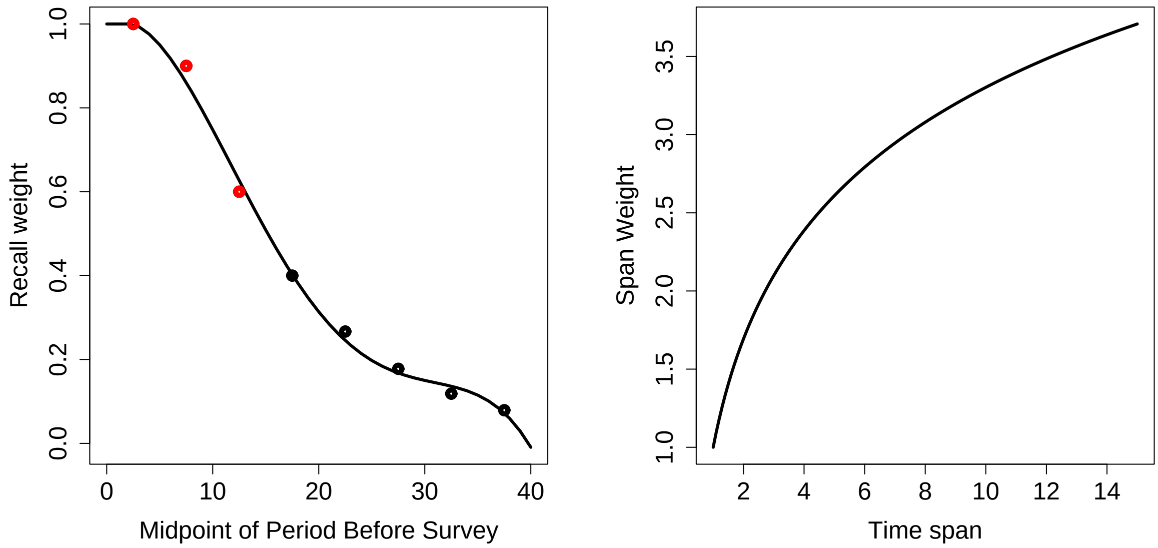


Figure 4: (a) Weights for the midpoint of the period before the survey (as given by a 4rth order polynomial function with recall weight constant at 1 till 2.5 years before the survey). The red dots are the rescaled weights given by Hill et al. (1998), the black dots are derived using the ratio of weights for the 7.5 years versus 12.5 years recall period. (b) Weights for time span.

With such a multiplier the empirical standard deviation of the measurement errors is equal to:

$$\sigma_{cts} = M_{cts}\delta_c, \tag{15}$$

with δ_c reflecting the overall data quality/standard deviation of the measurement errors in a country. Note that the data quality multiplier increases as data quality decreases, such that the error variance increases. The multiplier M_{cts} for data quality of observation y_{cts} is a function of weight w_{cts}. For a known outcome of the TFR f_{ct}, M_{cts} can be examined as follows: First note that with (2) and (15) the expected value of the absolute difference between an observation and the TFR is proportional to:

$$E|y_{cts} - f_{ct}| \propto M_{cts}\delta_c. \tag{16}$$

Define the absolute standardized residual r_{cts} as the absolute difference between true and observed TFR, taking into account the variability of the data in the country:

$$r_{cts} = \frac{|y_{cts} - f_{ct}|}{\delta_c}. \tag{17}$$

From (16) and (17) it follows that the expected value of r_{cts} is approximately proportional to M_{cts}:

$$Er_{cts} \propto M_{cts}. \tag{18}$$

To examine M_{cts}, the absolute standardized residuals were calculated based on estimates for the TFR for several countries (the TFR estimates were based on the UN WPP 2004 estimates and preliminary results of our model). The residuals are plotted versus the weights in Figure 5. If we choose M_{cts} to be a function of the weights that is close to the loess smoother (shown in red), then $Er_{cts} \approx M_{cts}$, such that M_{cts} reflects the difference in standard deviation between measurement errors. The black line is a least squares fit of a linear function to the data points and gives the multiplier M_{cts} as a function of the weights.

Confidence intervals for the TFR for each observation are constructed using the expression for the multiplier and an estimate of the baseline country-specific error variance, as shown for India and Senegal in Figure 6. Data quality is better in India which gives smaller confidence intervals for the TFR than in Senegal. The observations that are further away from the general trend are more likely to be of worst quality and are more likely to have a larger data quality multiplier, thus give a larger confidence interval for the TFR.

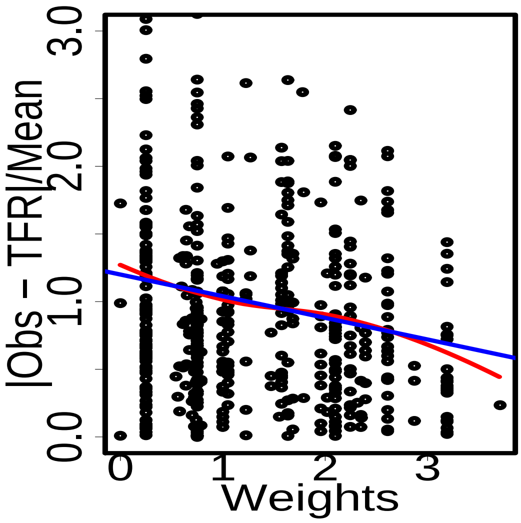


Figure 5: Absolute standardized residuals (which are proportional to the standard deviation of the measurement errors) versus the weights, with loess smoother (red) and the data quality multiplier M_{cts} (blue).

A potential criticism on this approach is the arbitrariness of the weighting scheme, eg. setting the weights for the source of the observation. We compared the results of the weighting scheme approach with a data-driven approach for selected countries (details not included here). In the data-driven approach we estimated the multiplier M_{cts} as a function of each of the data quality covariates. The results for both approaches were relatively similar which gives confidence in the weighting scheme. Moreover, the multiplier as given by the data-driven approach gave slightly worse estimates of the standard deviations of the measurement errors than than the multiplier based on the weighting scheme. Until a more complete data set covering more countries and types of data sources becomes available, the weighting scheme is chosen for modeling the difference in data quality.

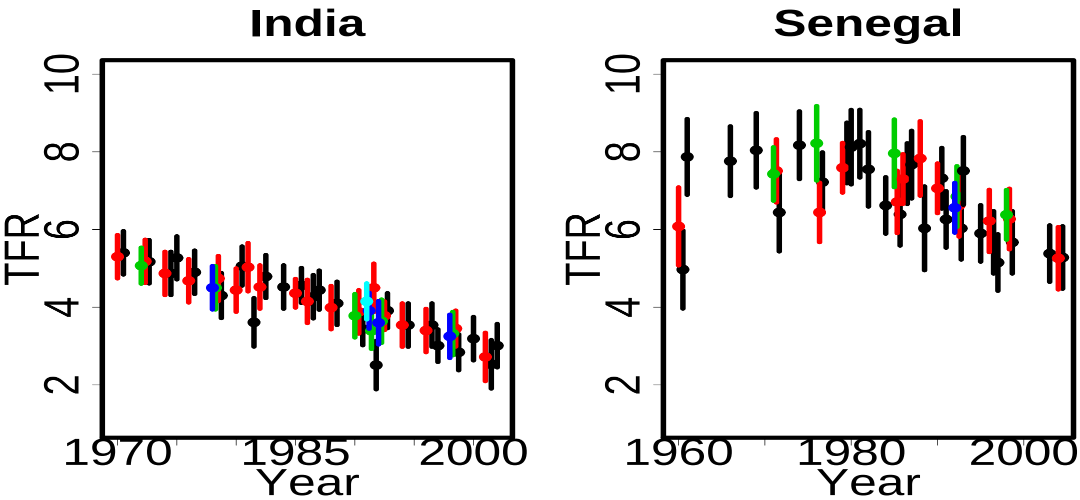


Figure 6: Difference in data quality; Each vertical line represents the 95% confidence interval for the TFR based on one observation, with the length of the interval determined by the data quality multiplier. (The colors are used to distinguish between the intervals, the observations in the same year have been drawn apart for the same reason.)

5 RESULTS

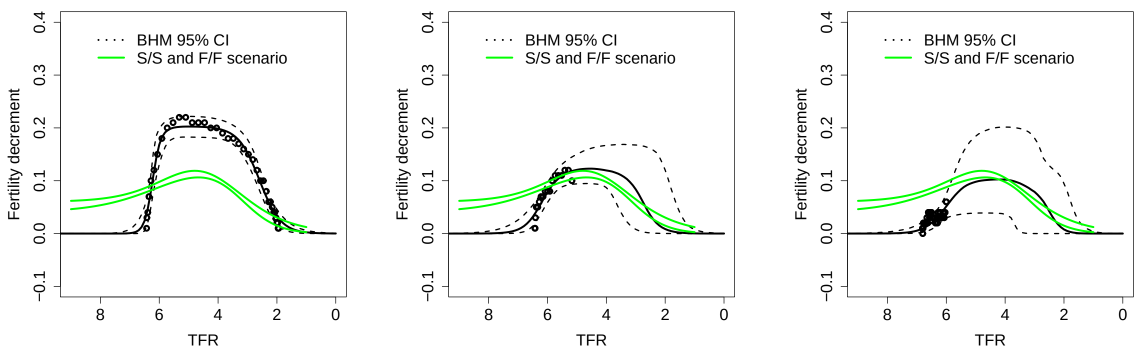
In the first part of this section we discuss the TFR predictions when applying a Bayesian hierarchical model to the estimates of the TFR as given by the UN World Population Projections, the 2004 revision (United Nations, Department of Economic and Social Affairs, Population Division 2005). This illustrates the uncertainty in the pace of the future decline, ignoring the uncertainty in the estimates of the past. In the second part we will show the preliminary results when applying the model to the raw data for some selected countries.

5.1 Predicting fertility based on UN estimates

The "data sources" in this section are the annual estimates of the TFR for all countries, as given in the UN World Population Prospects, the 2004 Revision (United Nations, Department of Economic and Social Affairs, Population Division 2005). For each country, only the period for which the estimates are based on empirical data is considered. The Bayesian hierarchical model us used to derive the country-specific projections.

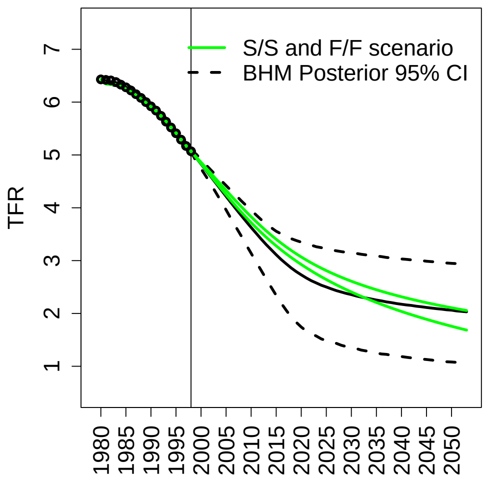
Figure 7 shows the confidence intervals for the pace of the fertility decline in Thailand, Cameroon and Madagascar. The uncertainty in the pace of the fertility decline differs substantively between the three countries. In Thailand the fertility transition has been completed, there is little uncertainty about the pace in the past decline. In Cameroon the TFR is around 5 children, there is considerable uncertainty about the future pace of the decline. For Madagascar, the uncertainty is even bigger, as the TFR is still around 6 children for each woman.

Figure 8 shows the prediction intervals for future TFR for Cameroon and Madagascar. The prediction interval for 2050 for Madagascar is wider than for Cameroon, for both countries the uncertainty is larger than one child. For Madagascar, the prediction interval is not symmetric around the median prediction, there is more uncertainty towards higher TFR.

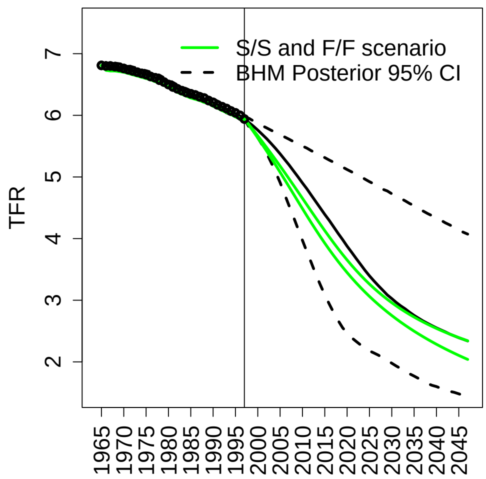


(a) Thailand (b) Cameroon (c) Madagascar

Figure 7: 95% Confidence intervals for the country-specific decline curves for (a) Thailand, (b) Cameroon and (c) Madagascar. The black dots are the annual decrements as given by the UN WPP 2004 estimates, the Fast/Fast and Slow/Slow trajectories are shown in green.



(a) Cameroon



(b) Madagascar

Figure 8: Predictions for (a) Cameroon and (b) Madagascar; Median TFR (solid line) and the 95% prediction intervals (dashed lines). The Fast/Fast and Slow/Slow trajectories are shown in green, the black dots are the UN estimates.

5.2 Uncertainty in TFR estimates and projections

This section discusses the preliminary results of the uncertainty assessment of TFR estimates and projections for 12 countries in Asia and Africa (Bangladesh, Burkina Faso, Bhutan, Gambia, Guinea, India, Laos, Mali, Mauritania, Niger, Senegal and Thailand). The empirical data set for each country contains unadjusted and adjusted fertility rates from different sources, retrospective periods and estimation methods (direct, indirect).

The plots in Figure 9 show the 95% quantile based confidence intervals for the TFR in Bangladesh, Burkina Faso and Guinea (represented by the vertical lines). The middle black line is the the median TFR for each year and the observations are shown in green. The TFR as given by the double logistic curve is shown in red in the background, most often it is not visible because it matches up with the median TFR. Note the difference in the size of the confidence intervals between the three countries, Bangladesh with relatively many data points that are close together has small confidence intervals, while Guinea with fewer data points and more difference between them has larger confidence intervals.

Figure 10 shows the projections up to 2050 for Bangladesh and Gambia. The UN Population Division chooses one out of three trajectories to predict fertility decline, the Slow/Slow and Fast/Fast trajectories are shown in turquoise. The high and low variant of the UN projections are shown based on the Fast/Slow trajectory in blue. In Gambia the TFR is most likely to decrease, but the data do not exclude the possibility that the fertility decline has not started yet, which results in wide prediction intervals.

In Figure 11 we check the validity of our model; the results are shown for Bangladesh and Bhutan. Each plot shows one trajectory of the TFR, with the 95% confidence interval for observed TFR. The observed data (plotted in green) is within its confidence bounds, and simulated data (plotted in purple) looks similar to the observations, which gives confidence in the modeling assumptions.

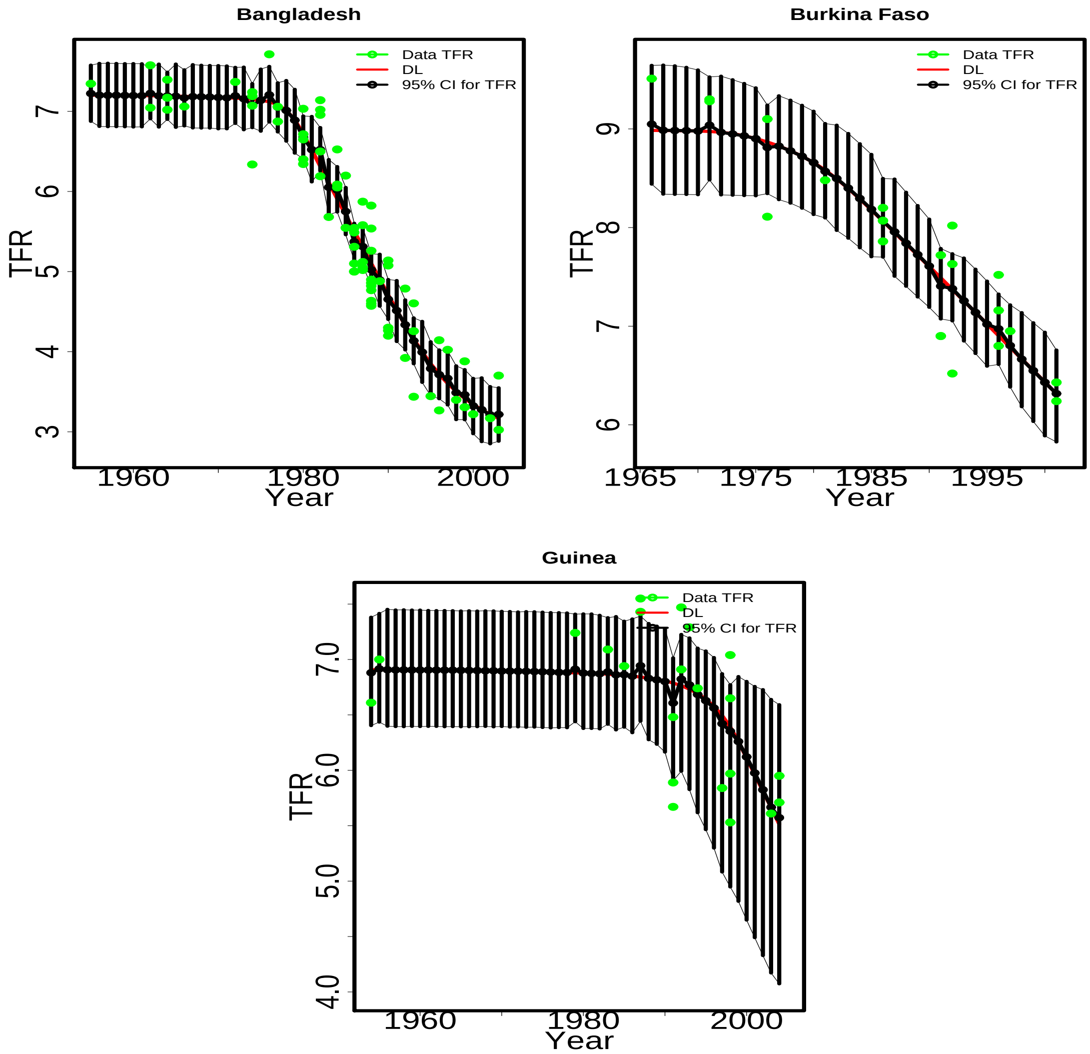


Figure 9: Uncertainty around estimated TFR for Bangladesh, Burkina Faso and Guinea. The solid black line is the median of the posterior sample for the TFR, the red line the median of the posterior sample for the theoretical TFR as given by the double logistic function (often not visible because it matches up with the true TFR). The observations are shown in green and the vertical lines represent the quantile-based 95% confidence bounds.

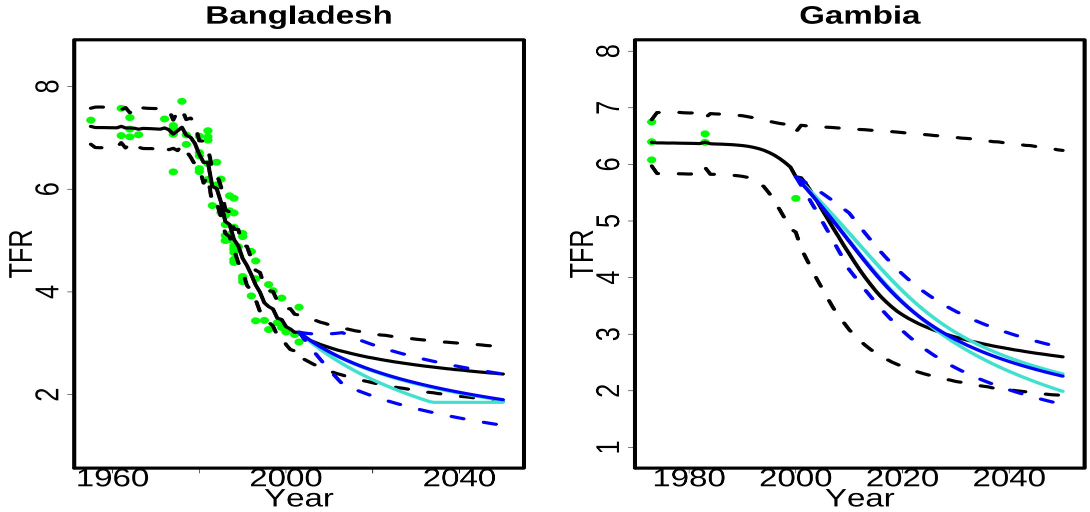


Figure 10: Median projected TFR (black, solid line) and 95% quantile based prediction interval (black, dashed lines). The Slow/Slow and Fast/Fast UN trajectories are shown in turquoise. The Fast/Slow trajectory is shown in blue, the high and low variant for this trajectory are represented by the blue dashed lines.

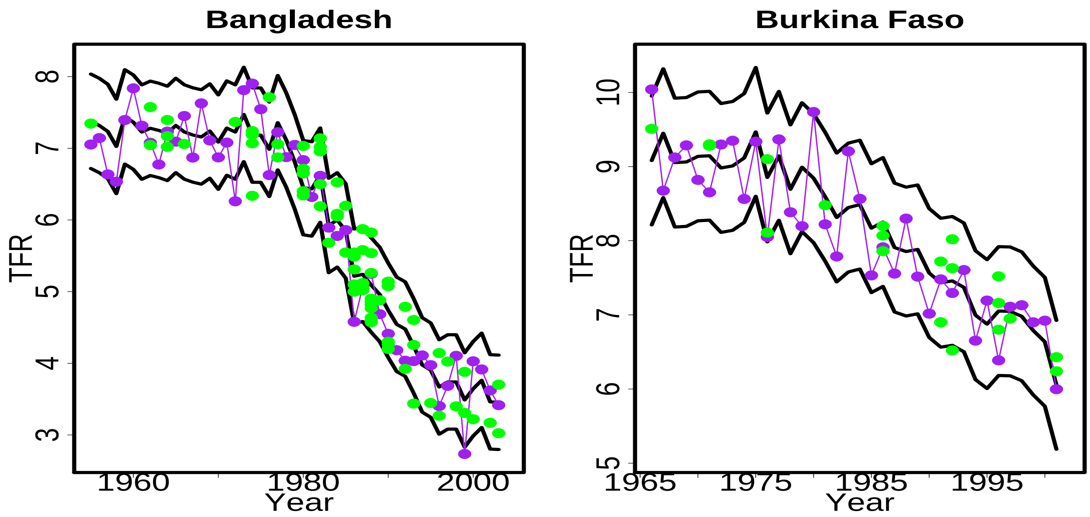


Figure 11: One trajectory of the posterior TFR and 95% confidence interval for observed TFR (black), with observations (green) and simulated observations (purple).

6 DISCUSSION

In this paper we propose a Bayesian hierarchical model to obtain probabilistic estimates and projections of total fertility rates. Differences in data quality for observations within a country are assessed by relating a standardized weighting scheme to the empirical variance of the measurement errors. We quantify the country-specific uncertainty in future fertility, as well as in the estimates of past levels of fertility. The preliminary results show the significant amount of uncertainty in past and future TFR levels, and the difference between various countries.

In order to produce projections of the population counts by age and sex, age specific fertility rates are needed. An extension to the model for the TFR as discussed here is to decompose the total fertility rate into age specific fertility rates, and to take into account the data on age specific rates. Combining data on fertility with population counts and mortality rates in the cohort component projection model allows for following cohorts over time and can reduce the uncertainty in each of the components.

The model can be extended for countries with a stalled fertility decline, by allowing for autocorrelation of the distortion terms.

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