REVISED DRAFT FOR PAA 2008
4 March 2008

THE COMPRESSION OF DEATHS ABOVE THE MODE

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ABSTRACT

The frequency distribution of ages at death has been shifting to the right, but it has not retained exactly the same shape. The ages of deaths above the mode have become more compressed. The paper investigates the reasons for this phenomenon. One of the simple models of mortality is found to be appropriate and mathematically tractable. Changes in the modal age of death and in the compression are shown to be driven by the way in which age-specific death rates fall at ages 70 and over. Both can be predicted from the death rates. Results are illustrated by data from the English Life Tables and Interim Life Tables, and these are confirmed by extensive data for six countries (including England and Wales) using the Human Mortality Database. Amongst other things, the paper illustrates how it is possible for the slope of the mortality curve to steepen while people are living longer, thus implying that the traditional ageing rate is not a valid measure of senescence.

BACKGROUND

As death rates fall and people live longer, the frequency distribution of the ages at death shifts to the right. However, when it does this, the distribution does not retain exactly the same shape. Kannisto (2001) presented extensive evidence to show that it was not simply sliding to the right. Instead, the right hand slope was being flattened vertically, so that the distribution became more compressed, as if (in his words) it was meeting an invisible wall. He contended that the ascending trajectory of mortality at high ages formed such a barrier but only in a relative sense, offering increasing resistance to further progress without setting any definite limit to it.

Kannisto's method of analysing this problem was to calculate the modal age of death M, the expectation of life at the mode, denoted by e(M), and the standard deviation (root mean square) of those individual life deviations from the mode which were positive. This upward standard deviation is denoted by SD(M+). He found that in cases where M had risen, there had generally been a fall in both e(M) and SD(M+). Since these are measures of dispersion, their fall showed that the ages at death above the mode had become more compressed. He showed that this compression had occurred in four countries with data back to the 19^{th} century and in 13 countries between 1960 and 1995, though it should be mentioned that in all these cases he only analysed the data for females.

Several questions arise. What are the circumstances in which compression will occur? Are the falls in e(M) and SD(M+) directly related to the rises in M? Are they inevitable if M rises, or can they be avoided? Can compression occur independently of a rise in M? Is compression a permanent feature? Are there any implications for future projections of old age mortality and the human life span?

Kannisto's work has been quoted as a reference in at least 25 papers, but so far as is known, the questions in the previous paragraph have not yet been settled. In order to answer them, we shall find it helpful to make use of a simple working model of old-age mortality, which will provide a clearer understanding of how the "invisible wall" operates.

We shall begin with the methodology, by explaining the reasons for the choice of the model. The main results will then be illustrated by Figures and numerical examples.

METHODOLOGY

Choice of model

We begin with a simple statement of fact. In a life table, the numbers of deaths at age x, denoted by d(x), are calculated from the observed or assumed values of the death rates at each age. The values of M and e(M) and SD(M+) are then calculated from the d(x). If the death rates do not change, then M and e(M) and SD(M+) will not change. If the death rates change, then so will M and e(M) and SD(M+). Accordingly, all changes in the mode and the whole development of compression must depend entirely on the way the death rates change. Although logically this must obviously be true, it is so general a statement that it does not do much to help us to understand how the process of compression works. What are the conditions in which compression will occur?

In order to consider this question, it is very helpful to have a simple working model of old-age mortality. Even if this model does not fit the data absolutely precisely, it will still give us a reasonable idea of what we can expect to happen, to the mode and to compression, when the death rates change in certain ways. Any departures from the model can also be informative.

The choice of a model is an important decision which needs to be described fairly fully. We are looking for a model which will fit the data at high ages reasonably well and which will be simple enough to provide the answers to our questions.

There are two main contenders for this role. The first is the model originally proposed by Lexis, under which deaths are regarded as either normal or premature. The normal deaths are presumed to produce a normal distribution for the ages of death. Nearly all deaths above the mode can reasonably be regarded as normal rather than premature. Accordingly, the distribution of the ages at death above the mode will be the upper half of a normal distribution, with mode M and standard deviation SD(M+). Despite having only two parameters, this fits the data on d(x) at high ages very well. However, it is technically difficult to fit unless the normal curve is extended for a few years below the mode. This normal extension of Lexis has been used by Cheung and Robine (2007), and Robine, Cheung, Thatcher and Horiuchi (2006). However, one must not extend the normal distribution too far, because premature deaths will alter the shape.

The second contender is a special case of the logistic model of mortality, which also has a long history, with a considerable literature (see Thatcher, 1999). This special case also has only two parameters, and it is usually written in the form

$$\mu_x = a\, e^{bx} / (1 + a\, e^{bx}) \qquad (1)$$

Here μ_x is the force of mortality at age x, while a and b are parameters which are constant in any given period.

The numerator in (1) will be recognised as Gompertz's "law of mortality", in which death rates increase exponentially with age as the body deteriorates. The denominator then converts (1) from an exponential into a logistic function, the rationale for this conversion being that people have different "frailties" and that it is the fittest who will survive to reach the highest ages.

The simple logistic model is less familiar than the normal distribution used by Lexis, so we need to examine its properties. It has only two parameters, whereas the general logistic model has four. One of these (Makeham's constant) becomes significant at some stage below age 70, while the other allows for alternative limits of mortality at extremely high ages, such as supercentenarians. For the present paper, however, we are not concerned with ages below 70 or with the most extreme high ages. Within the range to which we shall confine ourselves, the simple 2-parameter version seems to be adequate.

The model (1) can be written in a simplified form, if we make use of the mathematical function known as the logit function, which is only the difference between two logarithms. The logit function is defined by

$$\text{logit}(z) = \ln(z/(1-z)) = \ln z - \ln(1-z) \qquad (2)$$

With this notation it is easily seen that (1) can be written as

$$\text{logit}(\mu_x) = a^* + bx \qquad (3)$$

where $a^* = \ln a$. Thus if we calculate $y = \text{logit}(\mu_x)$ then the points (x, y) will lie on (3), which is a straight line. We shall call this the "logit line".

The fact that this very special case of the logistic model holds approximately, at least for modern data at high ages, was first noted by Kannisto (1992). He had plotted values of logit(μ_x) and saw that they were on a straight line. It was also used independently by Himes, Preston and Condran (1994). It was one of the models which were fitted by Thatcher, Kannisto and Vaupel (1998) in their exhaustive analysis of the Kannisto-Thatcher data base, which was the largest assemblage of official data on old-age mortality available at the time, covering 14 countries with reliable data..

The model (1) has a very important property, crucial to the study of compression. Although the death rates at individual ages depend on both of the parameters a and b, the compression (as measured by e(M) and SD(M+) depends only on the single parameter b. The reason for this can be explained mathematically as follows. It can be shown that when μ_x follows (1) then the modal age of death will occur at the age M which satisfies

$$ae^{bM} = b \qquad (4)$$

It then follows that (1) can be written as

$$\mu_x = be^{bX} / (1 + be^{bX}) \qquad (5)$$

where $X = x - M$, which is the age measured from the mode. The significance of this is that if we measure ages from the mode, then the whole shape of the distribution of both death rates and of ages at death depends only on the single parameter b.

Of course, it is not the parameter b which determines the death rates. It is the death rates which determine the parameter b. Nevertheless it is a very convenient property that in the simple logistic model the single parameter b is enough to summarise all we need to know about the distribution of the ages at death measured from the mode at a given moment of time.

Thus in the simple logistic model the compression of the ages at death above the mode, as measured by either e(M) or by SD(M+), can be calculated from b by using (5) to produce a life table starting from the mode. The numerical relationship is shown in Table 1. If we know b, then we can simply read off the values of e(M) and SD(M+). The table also shows that in the simple logistic model the ratio of SD(M+) to e(M) falls in a very narrow band, between 1.231 and 1.235. We recall that Kannisto found that the ratios he calculated from the original data for d(x) were hardly ever outside the range from 1.23 to 1.25. In the Lexis model, the ratio is always exactly 1.253, because this is the ratio of the standard deviation to the mean deviation in a normal distribution. In view of this relationship of direct proportionality, results for e(M) will always imply results for SD(M+), and vice versa.

A further important feature of the model is that while b determines the compression, as measured by e(M) and SD(M+), the two values a and b in conjunction enable us to calculate the mode M. From equation (4) it follows that

$$M = (\ln b)/b - (\ln a)/b \qquad (6)$$

Thus the compression is determined by the slope of the line (5), while the mode depends on the level as well as the slope.

Choice between the models

If we have to choose between using the normal extension of Lexis and the simple logistic model, the choice is fortunately made much less critical by the fact that these two models produce estimates of the distribution of deaths d(x) which are remarkably close.

The conditions in which the Lexis and logistic models can produce almost the same pattern of d(x) above the mode have been investigated by Horiuchi, using a 3-parameter version of the logistic model which includes the simple logistic as a special case. His first results, in Robine, Cheung, Thatcher and Horiuchi (2006), show that the relative rate of decrease of d(x) with respect to age, above the mode, is a linear function of age in the Lexis model, but a logistic function of age in the logistic model. However, any logistic function is almost linear in the neighbourhood of its point of inflection. As a consequence, there is a considerable range of ages above the mode where the two rates of decrease of d(x) are quite close. This is an important step towards understanding the otherwise puzzling similarity between the two models.

For the purpose of the present paper, the mathematical simplicity of (5) and (6) is a great advantage, so we shall adopt the simple logistic model, but fortunately the closeness between the models means that there is no reason to suppose that the Lexis model would have produced any different conclusions about compression.

Practical application

In order to apply the simple logistic model, we need to be able to estimate the parameter b. Since b is the slope of the straight line (3), it is sufficient to know the value of μ_x at any two ages, say x_1 and x_2, where $x_2 > x_1$. The slope of the line between these ages is then given by

$$b = [\text{logit}\ \mu(x_2) - \text{logit}\ \mu(x_1)] / (x_2 - x_1) \qquad (7)$$

However, since the central death rate m(x) at age x satisfies the approximation

$$m(x) \approx \mu(x + ½) \qquad (8)$$

we can easily show that

$$b \approx [\text{logit}\ m(x_2) - \text{logit}\ m(x_1)] / (x_2 - x_1) \qquad (9)$$

which can readily be calculated from life tables.

There is a wide choice for the ages x_1 and x_2. In theory, in a range where the data fit the simple logistic model precisely, we could choose any pair of ages whatever and they would all produce exactly the same value for b. However, we cannot take x_1 below 70, because the simple logistic model may no longer apply. Also, because observed death rates have standard errors, the standard error of b will be smallest if x_1 and x_2 are separated as widely as possible, provided that x_2 is not so high that the numbers of deaths are small. Most of the calculations in this paper take x_1 as 70 and x_2 as 90, so that b is estimated as the slope of the logit line between ages 70 and 90. Later, this slope will be compared with the slope between ages 80 and 90, using extensive data from six countries. To anticipate, the differences are generally small.

By comparing the estimates of b at two different dates, we can see whether b is increasing, and hence whether compression is occurring. The situation is illustrated in the schematic Figure 5, by taking the y-axis as y = logit m(x). If the logits fall more at age 1 than at age 2, then it will be seen that the slope of the line will necessarily increase. If, for simplicity, we take the two ages as 70 and 90, we have a conclusion which is worth numbering:

If logit m(x) falls faster at age 70 than at age 90, then b will increase and compression will occur (10)

Of course, in the simple logistic model it is not possible for the death rates at ages 70 and 90 to change in isolation. Death rates at all the other ages have to change too, if the model is to be maintained. It would be more accurate to say that compression will occur if the death rates at ages 70 and over follow the simple logistic model and change in such a way that logit m(x) falls faster at age 70 than at age 90.

Predicting e(M), SD(M+) and M

If we know the value of the parameter b, we can predict the values which we shall find for e(M) and SD(M+), by interpolating in Table 1.

The mode M can be predicted from the equation (6). Just as we found the parameter b from the slope of the line (3), so we can find the parameter a from the intercept. On substituting in (6) and using the approximation (8) we obtain

$$M = (\ln b)/b - [\text{logit } m(x_1)]/b + x_1 + \tfrac{1}{2} \qquad (11)$$

The relationship between b and the ageing rate

The parameter b is the rate at which logit m(x) increases with age, and so is the slope of the logit line y = logit m(x). This must not be confused with the slope of the mortality curve y = ln m(x). This second slope, the relative rate at which death rates increase with age at a given time, is known as the lifetime ageing rate (LAR). It is sometimes used as a measure of a hypothetical "rate of ageing". It can be calculated either as an average ageing rate between two fixed ages, or as the ageing rate at a given fixed age x. In general, the LAR at a fixed age will depend

on the age and so can be written as LAR(x). In geometrical terms, LAR(x) is the slope of the tangent to the mortality curve
y = ln m(x) at the age x.

In the simple logistic model, at a given time, b is the same at all ages but LAR(x) depends on the age. The relationship between the two is

$$\mathrm{LAR}(x) = b(1 - m(x)) \qquad (12)$$

This relationship follows from the fact that in the simple logistic model $(d\mu/dx)/\mu = b(1 - \mu)$. Thus at any age where we know m(x) it is possible to calculate LAR(x) from b and also b from LAR(x).

We may also note several consequences of (12). Firstly, at a given time, m(x) will increase with age and so (1 – m(x)) will fall. Hence LAR(x) will fall with age.

Next, we may consider changes over time. If mortality is falling generally, then at a given fixed age m(x) will be falling and (1 – m(x)) will be rising. If there is no compression, so that b remains constant, then it follows from (12) that LAR(x) will be rising, at any given fixed age. With no compression, the distribution of ages at death will be shifting to the right without any change of shape, so that the LAR at a given age will be succeeded by a LAR which was previously at a younger age, and therefore higher.

Next, if there is compression while mortality is falling, then both the factors on the right hand side of (12) will be rising at the same time. Thus LAR(x) will necessarily rise at any given fixed age, by more than it would have done if there had been no compression.

Finally, there is the case where b and m(x) are both falling. It is then possible that LAR(x) may either rise or fall, depending on the sizes of the changes in b and m(x).

The ageing rate and the length of life

A perhaps unexpected relationship between the ageing rate and the length of life can also be seen from the schematic diagram (Figure 5), by taking the y-axis as y = ln m(x), so that we are looking at the mortality curve. The upper straight line is the chord which joins the two points at age 1 and at age 2, both at time 1. The slope of this line is then the lifetable ageing rate (LAR) at time 1. The lower straight line shows the position at time 2. The schematic diagram illustrates a case where mortality has fallen, but has fallen more at age 1 than at age 2. The slope of the lower line is then steeper than the slope of the upper line, so the mortality curve has steepened and the ageing rate has risen. At the same time, because mortality has fallen at all ages in the range, people will be living longer. This apparently paradoxical result presumably implies that the traditional ageing rate is not a valid measure of senescence.

ILLUSTRATIONS FOR ENGLAND AND WALES

The illustrations which follow use data for England and Wales, obtained from the English Life Tables and Interim Life Tables. They concentrate on the ages from 70 to 95, because it is the death rates in this range which determine the modal age of death and also dominate the expectation of life at the mode.

In the simple logistic model, the points $y = \text{logit}(m_x)$ lie on the straight lines (3), so our first step is to see how close the observed points are to straight lines. Figures 1 and 2 accordingly plot the observed points for males and females at the three widely spaced dates 1906, 1971 and 2004. (The number 10 has been added to all the logits in order to avoid negative numbers, but this does not affect the shape of the lines). It can readily be seen that the observed points look reasonably like straight lines. It must be remembered that we are not formally fitting straight lines or testing an hypothesis. We are only seeking confirmation that the observed points are close enough to straight lines to justify the choice of the logistic as a simple working model, which will help to interpret results. As we can see by eye, the lines do not all have the same slopes. The model then indicates that there were changes over time in the parameter b and hence in e(M) and SD(M+). We shall quantify these shortly.

Figures 3 and 4 show how the modal ages of death have varied. The full lines in Figure 3 show the observed values for males found from the life tables. There was little change between 1841 and 1906, but the mode then started to rise, rather falteringly. A strong rise did not start until 1971, but then it was faster than the rise for females, and this rise continued at a rapid pace for thirty years. The picture for females in Figure 4 is notably different. Again there was little change between 1841 and 1906, but the mode then started to rise and has risen ever since.

The dashed lines on Figures 3 and 4 show the predictions of the mode which are given by the model, found by using the equation (11). These predictions were all made independently of each other, using only the observed death rates (and hence the logits) at ages 70 and 90. The predictions are quite close to the observed values. In comparing them it must be remembered that the mode M found from the life tables is rounded down to give a whole number for the age x. This will account for part of the differences between the two curves, which in any case is generally less than 12 months. This comparison may serve to reassure us that the model gives reasonable predictions, and also that the observed changes in the mode were reasonably close to what the model would lead us to expect, given what happened to the death rates at ages 70 and over.

The data plotted in Figures 3 and 4 are given in Table 2. This also shows the estimated values of the parameter b and the resulting predictions of e(M) and SD(M+) which are given by the model.

We now quantify the changes in compression which were indicated by the slopes of the lines in Figures 1 and 2, using for this purpose the data assembled in Table 3. These include a refinement. The letter M denotes the mode estimated as the age which gives the highest value for d(x) in the life table. The letter M* denotes the mode of the continuous curve of deaths, which may be up to a maximum of 12 months higher than M. M* is here estimated by the formula used by Kannisto (2001), which effectively approximates the tip of the continuous curve by the parabola which produces the correct observed values of d(x-1), d(x) and d(x+1). The expectation of life at M* is then found by interpolating between e(M) and e(M+1) in the life table. SD(M*+) then follows on multiplying by (say) 1.24.

It will be seen from Table 3 that for males, e(M*) and SD(M*) both rose and fell. For females, we see only falls and this is the progression which was observed by Kannisto (2001). The changes were not continuous, but over the period as a whole the expectation of life for females at the mode M* fell by 2.35 years and the standard deviation fell by 2.94years.

We may note that these changes in compression are consistent with the conclusion numbered (10) above, that compression can be predicted from the relative falls in the logits at ages 70 and 90. The relevant falls can be derived from Table 3:

	Falls in logit m(x)	
	Age 70	Age 90
Males from 1906 to 1971	0.220	0.350
Males from 1971 to 2004	0.836	0.398
Females from 1906 to 1971	0.753	0.405
Females from 1971to 2004	0.592	0.375

The first line stands out, as the case where the fall in the logits was less at age 70 than at age 90. When this happens, the theory predicts the opposite of compression: the change may not be uniform, but the standard deviation will be higher at the end of the period than at the beginning, as indeed happens (see Table 3).

RESULTS FOR SIX COUNTRIES

Logit lines for six countries

The logit lines in Figures 1 and 2, and the deductions and calculations which follow from them, are all based on data for England and Wales, taken from the English Life Tables and Interim Life Tables. It is natural to wonder whether similar results apply in other countries. A study has been made which covers five other countries using data from the Human Mortality Database (HMD), together with England and Wales from the HMD (for comparison with the English Life Tables and Interim Life Tables).

Logit lines (as in Figures 1 and 2) have been calculated and plotted for France (eight dates from 1899 to 2006), Italy (six dates from 1872 to 2003), Japan (five dates from 1947 to 2005), Sweden (seven dates from 1751 to 2005) and Switzerland (seven dates from 1876 to 2005), For England and Wales, the HMD data were plotted for seven dates from 1841 to 2003. For each country the logit lines were drawn for both males and females, making 40 lines in all. [These Figures will be made available on-line].

The main feature, which is immediately apparent from inspection, is that the great majority of the lines are practically straight. In some cases there are one or sometimes two wobbles, taking the form of deviations from the line in the same direction for a few years of age running. These wobbles do not have the appearance of random fluctuations. A possible explanation might be that in periods when death rates are falling, the falls may not happen absolutely simultaneously in all age groups every year. Be that as it may, the HMD logit lines show a few wobbles in England and Wales which are not apparent in the English Life Tables. This may be due to the fact that the English Life Tables (apart from 1841) are based on more than one year and are smoothed by actuarial techniques.

There are a few cases, though, where the departures from linearity are more severe. It is notable that these are mostly in either the first year of the series or in the last year. The first year cases include Italy in 1872, Sweden in 1751 and Switzerland in 1876. For England and Wales the 1841 line from the HMD is not very straight, though previous work had not found any problems for 1841 in the English Life Tables. However, in all these early years there may have been problems in collecting complete data, so deviations from straight lines are not surprising.

At the other extreme, in several countries (with the notable exceptions of Sweden and Switzerland) there are several slight departures from linearity at the latest dates, from 2003 to 2005. These again are not surprising, because the latest

published figures contain an element of estimation and are usually described as preliminary or interim. In fact, the national statistical systems provide for these figures to be revised as a matter of routine, as more data become available.
There is a wealth of material here which could be used for further study. In the present paper, though, we only have to decide whether to adopt the simple logistic as a working model, to help to explain how compression works.
Despite the reservations about the first and last dates, and in some cases a few wobbles in the middle, the work on the logit lines in the six countries confirms that the simple logistic model is a very reasonable choice for ages 70 and over.

The parameter b in the six countries

In the numerical examples for England and Wales, the parameter b was found from the slope of the logit line joining ages 70 and 90. An obvious question is what the b's look like in other countries, when fitted to data from the HMD.

The ages 70 and 90 were chosen with one below the mode and the other above it, and with a span of 20 years in order to minimise the standard errors of the estimates of the slope. A secondary question is whether these restrictions were necessary. If the data follow the simple logistic model perfectly, then any pair of ages would do, even if both are above the mode.

Table 4 provides the material to answer these questions. Columns (B), (C) and (D) Show the values of b given by the slopes at ages 70-80, 80-90 and 70-90. Column (G) shows the difference between the slopes at 80-90 and 70-90. If the logit line were perfectly straight, these differences would all be zero. As it is, they are mostly negligible. Any larger differences can only be due to the wobbles in some of the logit lines, as described above.

The columns (E) and (F) show the implications of the b's for compression. If b in column (C) has risen since the previous line, this is denoted in column (E) by the letter U (for up). This means that there was compression between the two dates. If it has fallen, this is shown by the letter D (for down). The U's and D's in column (F) show similarly the ups and downs in column (D). If the two letters on the same line are the same, this means that the slopes at ages 80-90 and 70-90 have changed in the same direction, and so tell the same story about what was happening to compression. Runs of U's downwards show continuing compression. The presence of the letter D shows that the rise in b has often been halted or even reversed, so that compression has not been a steady process. However, in all cases the b at the earliest date is lower than the b at the latest date, so the standard deviation was lower at the later date. Thus over the period as a whole there has been compression in all six countries, for both males and females.

Standard deviations in the six countries

Comparisons between the observed values of SD(M+) and the values predicted by the model, with parameters estimated from the death rates (and hence logits) at ages 70 and 90, have been made for all six countries for successive 5-year periods. The starting dates are 1850-4 for England and Wales and for Sweden, 1900-04 for France, 1875-79 for Italy,1950-54 for Japan and 1880-84 for Switzerland. The observed values of SD(M+) are standard deviations measured from the mode M taken as the age which has the highest d(x) in the life table. However, the model produces predicted values for SD(M*), where M* is the mode of the continuous curve of deaths. Thus a certain gap between the observed and predicted values has to be tolerated.

The results are given in a 6-page table which for reference is notionally numbered Table 5, though it is too large to be included in the present paper. [However, both the table and a 6-page set of Figures illustrating the results will be made available on-line]. The Figures illustrating Table 5 display the trends in the standard deviations, which often changed smoothly but sometimes not. The most exceptional case was for males in England and Wales, which as it happens has already been identified earlier in this paper from the English Life Tables. These males had a higher value of SD(M+) in 1971 than in 1906. The figures from the HMD trace the change in much more detail. Table 5 show that there was a long fall in SD(M+) from 1900-04 up to 1940-44, but this was then followed by a much larger rise up to 1970-74. After that, compression resumed. For males in Italy, there was a long steady fall in SD(M+) from 1900-04 up to 1960-64. Later there was a marked rise and so far compression has not resumed. Females in France showed a long steady fall up to 1980-84, which then levelled out and has so far remained steady. The case of the standard deviation in Japan has currently attracted attention, because it showed a fall up to the late 1980s and since then has been fairly steady. It will be seen from the other countries that levelling off is not new. There are precedents.

The present values of the observed standard deviation SD(M+) for males are 8.2 (years) in England and Wales, 6.9 in France, 8.1 in Italy, 7.6 in Japan, 7.7 in Switzerland and 7.0 in Sweden. For females they are 7.2 in England and Wales, 6.6 in France, 6.8 in Italy, 6.3 in Japan, 5.9 in Switzerland and 6.4 in Sweden.

Earlier research

It may be asked how this work relates to earlier results given by Strehler and Mildvan (1960) and by Gavrilov and Gavrilova (1991). Strehler and Mildvan wrote on the kinetics of death, using postulates about the distribution of stress magnitudes and the response of organisms. Their approach was based on the experimentally-determined Gompertz function. One of their important predictions was that there would be a negative correlation between the slope and the intercept of the Gompertz curve, so that the slope would tend to steepen as the level of mortality falls. This became known as the S-M correlation.

Gavrilov and Gavrilova, writing later, challenged the reliance on Gompertz. Instead they used the Makeham model and found that this greatly improved the fit and also affected the conclusions. Although the Makeham constant, representing premature mortality, is only a very simple device, at least it makes some allowance for deaths which are not associated with age. The correlation between slope and level can be affected by changes in premature mortality and is not just a feature of senescence. On fitting Makeham's law to mortality data from several different countries, they found that the fitted mortality curves gave the appearance that if they could only be continued further they would all be heading towards a single point, the same for all the countries, but unfortunately beyond the range of the observed data. The apparent tendency of the lines towards convergence implied a negative correlation between the slope and the intercept.

Whereas the above writers were concerned with ages up to 80, the simple logistic model comes into its own at ages over 80, where both the Gompertz law and the Makeham model cease to fit. Also, as we have made clear, the logit lines in the simple logistic model are not the same thing as the mortality curves used in Gompertz and Makeham. They have similarities at lower ages but then diverge. Nevertheless, despite these differences, it is of interest to note that Thatcher (1999), fitting logit lines to some historical life tables back to the 17th century, found that these lines had points of inflection which were very close to each other, so that they almost converged to a common point. The point of inflection is always at the age where $\mu_x = 0.5$, and the almost common point was between ages 98 and 100. This suggested that there was a very long period when death rates at very high ages were almost static, despite all the changes at lower ages. However, this static period started to come to an end in the 1950s, when death rates at very high ages started to fall. More recently Yashin et al (2001) have found that the S-M correlation was only stable in certain countries and that patterns could change, in ways which were not consistent with the S-M postulates.

Perhaps we might add the rather obvious point which is illustrated in Figure 5, showing that the slope of a mortality curve between any two fixed ages will necessarily increase if ln m(x) falls more at the lower age than at the higher age. This is a matter of geometry rather than biology. This seems to have produced the negative correlation between the slope and the level in the data used by Strehler and Mildvan and by Gavrilov and Gavrilova. The pronounced decline in infectious disease mortality in the early and middle twentieth century (the epidemiologic transition) lowered ln m(x) at younger adult ages more than those at older adult ages, making the slope steeper.

SUMMARY AND CONCLUSIONS

The aim of the paper is to study the reasons for the phenomenon reported by Kannisto (2001), that in many countries the expectation of life at the modal age of death (denoted by e(M)) and the standard deviation of the ages at death above the mode when measured upwards from the mode (denoted by SD(M+) have both been falling. This means that the ages at death which fall above the mode have been becoming more compressed.

In order to study this problem, we need a working model of mortality which fits the data reasonably well at ages 70 and over. The choice of model is an important decision and it is considered at some length. The choice falls on a very simple special case of the logistic model, which is remarkably close (above the mode) to a model originally proposed by Lexis, but has the advantage that it is mathematically more tractable. We may briefly summarise its main properties. It has two parameters, denoted by a and b. The parameter b on its own is sufficient to determine both e(M) and SD(M+), but the mode depends on both a and b. This shows that it is possible for the mode and the compression to change independently. They may often appear to be correlated, but there is no causal connection.

In theory, if the data follow the model exactly, both the parameters can be fitted if we know the death rates at just two arbitrary ages in the range 70 and over. (This gives two equations with two unknowns. The same is also true for the Lexis model, above the mode).

The model is initially tested, using as an example the parameters which fit the death rates at ages 70 and 90 taken from English Life Tables at the three widely-spaced dates 1906, 1971 and 2004. The results are successful. Also, we already find an example where the mode and the standard deviation both rose at the same time.

Using the same pair of ages, the model also shows that compression can be expected if the logit function of the death rates falls more at age 70 than at age 90. This provides a very simple method to detect or predict compression, if we know the death rates. It is also relevant to Kannisto's use of the term "invisible wall", to describe the force which appears to cause compression. He contended that the compression he observed was due to the resistance caused by the ascending trajectory of mortality. In our analysis, the compression was due to the fact that the death rates at very high ages did not fall faster than they did. This is a distinction without much of a difference.

We also give the relationship between the parameter b and the lifetime ageing rate (LAR).We show how the ageing rate can steepen while at the same time death rates are falling, so that people are living longer. This presumably means that the traditional ageing rate is not a valid measure of senescence.

The investigation is then extended to cover six countries, using data from the Human Mortality Database from very early dates to the present. Some of the results are given in Table 4, showing the values of the parameter b fitted both by starting from the pair of ages 70 and 90 and for comparison by starting from the ages 80 and 90.The letters U in the table identify periods when compression occurred in the country concerned. The presence of the letter D shows periods when the compression was halted or temporarily reversed. Comparing the earliest period with the last, there was compression in all six countries for both males and females.

Further extensive analyses for the six countries are described in the paper [and will be made available on-line]. They show, among many other things, that the standard deviation for males in England and Wales, after a very long fall, then rose for 30 years and has since been falling again. The standard deviation for females in France had a long fall but has since been level. The standard deviation for males in Italy had a long fall, followed by a temporary rise, and has since been level. Recent figures show that a levelling may now be happening for both males and females in Japan.

A final section describes earlier research by Strehler and Mildvan (1960) and by Gavrikov and Gavrilova (1991), who used the Gompertz and Makeham models. However, at the highest ages above the mode these models no longer fit and there may be other factors at work. The long-term average tendency for the logit function of the death rates to fall more at age 70 than at age 90 (for example) might perhaps reflect age-related changes in society. The growing numbers at age 90 and expensive treatments may be calling for relatively more provision of resources and relatively more carers than formerly. Some medical advances may be more effective at age 70 than at age 90. In order to account for a tendency towards compression it does not seem necessary to invoke any biological changes in the ageing process itself. How far compression can continue is an open question.

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TABLE 1

VALUES OF b, e(M) AND SD(M+)

b	e(M)	SD(M+)	Ratio SD(M+)/e(M)
0.09	7.34429078	9.038632	1.230702
0.092	7.2005943	8.862959	1.230865
0.094	7.06302674	8.694782	1.231028
0.096	6.93120507	8.533632	1.23119
0.098	6.80477744	8.379079	1.231352
0.1	6.68342016	8.230728	1.231514
0.102	6.56683494	8.088213	1.231676
0.104	6.45474646	7.951198	1.231837
0.106	6.34690016	7.819371	1.231998
0.108	6.24306029	7.692444	1.232159
0.11	6.1430083	7.57015	1.23232
0.112	6.04654127	7.452241	1.23248
0.114	5.95347044	7.338486	1.23264
0.116	5.86362015	7.228669	1.2328
0.118	5.77682663	7.122591	1.232959
0.12	5.69293696	7.020065	1.233118
0.122	5.61180833	6.920915	1.233277
0.124	5.53330713	6.824979	1.233436
0.126	5.45730819	6.732104	1.233594
0.128	5.38369419	6.642146	1.233752
0.13	5.31235506	6.55497	1.23391
0.132	5.24318737	6.470449	1.234068
0.134	5.17609382	6.388466	1.234225
0.136	5.11098292	6.308908	1.234382
0.138	5.04776843	6.231669	1.234539
0.14	4.98636909	6.15665	1.234696
0.142	4.92670812	6.083757	1.234852
0.144	4.8687131	6.012902	1.235009
0.146	4.81231554	5.944001	1.235164
0.148	4.75745066	5.876975	1.23532
0.15	4.70405713	5.811748	1.235476

TABLE 2

DATA PLOTTED IN
FIGS 3 AND 4

FIGURE 3

MALES IN ENGLAND AND WALES

Year	MODAL AGE OF DEATH Predicted	Observed	Parameter b	Predicted e(M)
1841	72.999	71.5	0.094307	7.04
1906	73.351	72	0.098928	6.75
1921	74.434	74	0.096267	6.91
1931	74.7	74	0.101799	6.58
1951	75.651	75	0.107409	6.27
1961	75.304	75	0.096639	6.89
1971	75.047	74	0.09189	7.21
1981	77.177	77	0.0962	6.92
1991	79.341	80	0.099906	6.89
2001	82.389	84	0.109159	6.18
2004	83.401	83	0.113777	5.96

FIGURE 4

FEMALES IN ENGLAND AND WALES

Year	MODAL AGE OF DEATH Predicted	Observed	Parameter b	Predicted e(M)
1841	74.307	73	0.095701	6.95
1906	75.14	74	0.096494	6.9
1921	77.463	77	0.100735	6.64
1931	78.05	79	0.107003	6.29
1951	80.278	80	0.116308	5.85
1961	81.446	81	0.116333	5.85
1971	82.495	82	0.113851	5.96
1981	83.618	84	0.1157	5.88
1991	84.748	86	0.112585	6.02
2001	86.36	86	0.12056	5.67
2004	86.93	87	0.124741	5.5

New Table 3

TABLE 3

DATA FOR 1906, 1971 AND 2004

	MALES				FEMALES			
Year	M	M*	e(M*)	SD(M*+)	M	M*	e(M*)	SD(M*)
1906	72	72.76	7.24	8.98	74	74.02	7.48	9.28
1971	74	74.62	7.47	9.26	82	82.42	5.97	7.4
2004	83	83.97	6.12	7.59	87	87.76	5.13	6.36

	MALES				FEMALES			
Year	m(70)	logit m70	m(90)	logit m90	m(70)	logit m70	m(90)	logit m90
1906	0.0694	-2.5854	0.3505	-0.6169	0.0581	-2.786	0.2981	-0.8561
1971	0.5706	-2.805	0.2755	-0.9671	0.0282	-3.5385	0.2207	-1.2615
2004	0.0255	-3.6413	0.2033	-1.3658	0.0158	-4.131	0.163	-1.6363

PARAMETER b

	MALES	FEMALES
1906	0.0989	0.0965
1971	0.0919	0.1139
2004	0.1138	0.1247

DEFINITIONS AND METHOD

M is the age with the highest d(x) in the life table
M* is the estimated mode of the continuous curve of deaths
e(M*) is interpolated between e(M) and e(M+1)
SD(M*+) is e(M*) multiplied by 1.24

SOURCES

1906 from English Life Table for 1901-1910
1970 from English Life Table for 1970-72
2004 from Interim Life Table for England and Wales 2003-05

TABLE 4

ESTIMATES OF THE PARAMETER b

(A) Date	(B) Slope at 70-80	(C) Slope at 80-90	(D) Slope at 70-90	(E) up/down (C)	(F) up/down (D)	(G) C - D	(H) (B+C)/2
ENGLAND AND WALES FROM ELTs							
Females							
1906	0.090893	0.102097	0.096495			0.005602	0.096495
1971	0.114328	0.113374	0.113851	U	U	-0.000477	0.113851
2004	0.121046	0.128174	0.12461	U	U	0.003564	0.124610
Males							
1906	0.088199	0.109658	0.098929			0.010729	0.098929
1971	0.088663	0.095118	0.09189	D	D	0.003228	0.091891
2004	0.114171	0.112954	0.113563	U	U	-0.000609	0.113563
ENGLAND AND WALES FROM HMD							
Females							
1841	0.081627	0.092839	0.087233			0.005606	0.087233
1906	0.09513	0.096742	0.095936	U	U	0.000806	0.095936
1951	0.120182	0.111218	0.1157	U	U	-0.004482	0.115700
1971	0.115476	0.114045	0.114761	U	D	-0.000716	0.114761
1981	0.113908	0.113336	0.113622	D	D	-0.000286	0.113622
1991	0.105245	0.12039	0.112817	U	D	0.007573	0.112818
2003	0.121174	0.129178	0.125176	U	U	0.004002	0.125176
Males							
1841	0.08416	0.094032	0.089096			0.004936	0.089096
1906	0.088878	0.101213	0.095045	U	U	0.006168	0.095046
1951	0.102305	0.113922	0.108114	U	U	0.005808	0.108114
1971	0.089021	0.093125	0.091073	D	D	0.002052	0.091073
1981	0.099089	0.096008	0.097549	U	U	-0.001541	0.097549
1991	0.097865	0.107842	0.102853	U	U	0.004989	0.102854
2003	0.114554	0.114063	0.114308	U	U	-0.000245	0.114309
JAPAN							
Females							
1947	0.10738	0.107096	0.107238			-0.000142	0.107238
1971	0.126498	0.120281	0.123389	U	U	-0.003109	0.123389
1981	0.135889	0.128203	0.132046	U	U	-0.003843	0.132046
1991	0.133621	0.145447	0.139534	U	U	0.005913	0.139534
2005	0.12213	0.141526	0.131828	D	D	0.009698	0.131828
Males							
1947	0.096421	0.102165	0.099293			0.002872	0.099293

1971	0.107872	0.107678	0.107775	U	U	-0.000097	0.107775
1981	0.116679	0.114437	0.115558	U	U	-0.001121	0.115558
1991	0.117913	0.116366	0.117139	U	U	-0.000774	0.117139
2005	0.109725	0.098083	0.103904	D	D	-0.005821	0.103904

ITALY

Females

1872	0.122811	0.106229	0.11452			-0.008291	0.114520
1906	0.092398	0.080278	0.086338	D	D	-0.006060	0.086338
1951	0.122655	0.113294	0.117975	U	U	-0.004680	0.117975
1971	0.135414	0.115464	0.125439	U	U	-0.009975	0.125439
1981	0.131821	0.128573	0.130197	U	U	-0.001624	0.130197
1991	0.123951	0.139266	0.131609	U	U	0.007658	0.131609
2003	0.129345	0.149533	0.139439	U	U	0.010094	0.139439

Males

1872	0.107759	0.094221	0.10099			-0.006769	0.100990
1906	0.100367	0.103561	0.101964	U	U	0.001597	0.101964
1951	0.116844	0.117483	0.117164	U	U	0.000319	0.117164
1971	0.103745	0.10213	0.102937	D	D	-0.000807	0.102937
1981	0.104089	0.105234	0.104662	U	U	0.000572	0.104662
1991	0.102499	0.116686	0.109593	U	U	0.007093	0.109593
2003	0.117475	0.119573	0.118524	U	U	0.001049	0.118524

FRANCE

Females

1899	0.107313	0.099771	0.103542			-0.003771	0.103542
1906	0.103801	0.108153	0.105977	U	U	0.002176	0.105977
1951	0.124867	0.119767	0.122317	U	U	-0.002550	0.122317
1971	0.126138	0.123387	0.124762	U	U	-0.001376	0.124762
1981	0.135173	0.130977	0.133075	U	U	-0.002098	0.133075
1991	0.128032	0.142547	0.13529	U	U	0.007258	0.135290
2005	0.12416	0.157302	0.140731	U	U	0.016571	0.140731

Males

1899	0.11155	0.099967	0.105758			-0.005791	0.105758
1906	0.104302	0.125879	0.11509	U	U	0.010788	0.115090
1951	0.109159	0.109941	0.10955	D	D	0.000391	0.109550
1971	0.093623	0.1061	0.099861	D	D	0.006239	0.099861
1981	0.108615	0.107037	0.107826	U	U	-0.000789	0.107826
1991	0.102727	0.118965	0.110846	U	U	0.008119	0.110846
2005	0.107494	0.130949	0.119222	U	U	0.011727	0.119222

Sweden

Females

	1751	0.078069	0.098516	0.088293			0.010223	0.088293
	1906	0.116966	0.129758	0.123362	U	U	0.006396	0.123362
	1951	0.120254	0.123901	0.122078	D	D	0.001823	0.122078
	1971	0.123721	0.122353	0.123037	D	U	-0.000684	0.123037
	1981	0.121867	0.133409	0.127638	U	U	0.005771	0.127638
	1991	0.126136	0.136997	0.131566	U	U	0.005431	0.131566
	2005	0.116947	0.149207	0.133077	U	U	0.016130	0.133077
Males								
	1751	0.072473	0.104483	0.088478			0.016005	0.088478
	1906	0.105517	0.100553	0.103035	D	U	-0.002482	0.103035
	1951	0.109916	0.131857	0.120887	U	U	0.010970	0.120887
	1971	0.106333	0.102723	0.104528	D	D	-0.001805	0.104528
	1981	0.107285	0.105325	0.106305	U	U	-0.000980	0.106305
	1991	0.111277	0.124089	0.117683	U	U	0.006406	0.117683
	2005	0.11815	0.137503	0.127826	U	U	0.009677	0.127826
Switzerland								
Females								
	1876	0.085875	0.098566	0.092221			0.006346	0.092221
	1906	0.095683	0.105135	0.100409	U	U	0.004726	0.100409
	1951	0.118221	0.12434	0.121281	U	U	0.003059	0.121281
	1971	0.127252	0.137785	0.132518	U	U	0.005267	0.132518
	1981	0.140368	0.130854	0.135611	U	U	-0.004757	0.135611
	1991	0.122241	0.151027	0.136634	U	U	0.014393	0.136634
	2005	0.118399	0.153591	0.135995	U	D	0.017596	0.135995
Males								
	1876	0.095251	0.106461	0.100856			0.005605	0.100856
	1906	0.098125	0.101114	0.09962	D	D	0.001495	0.099620
	1951	0.101976	0.119589	0.110782	U	U	0.008807	0.110782
	1971	0.107745	0.106407	0.107076	D	D	-0.000669	0.107076
	1981	0.106121	0.105729	0.105925	D	D	-0.000196	0.105925
	1991	0.106086	0.113549	0.109818	U	U	0.003731	0.109818
	2005	0.111846	0.13296	0.122403	D	U	0.010557	0.122403

FIGURE 1: MALES IN ENGLAND AND WALES

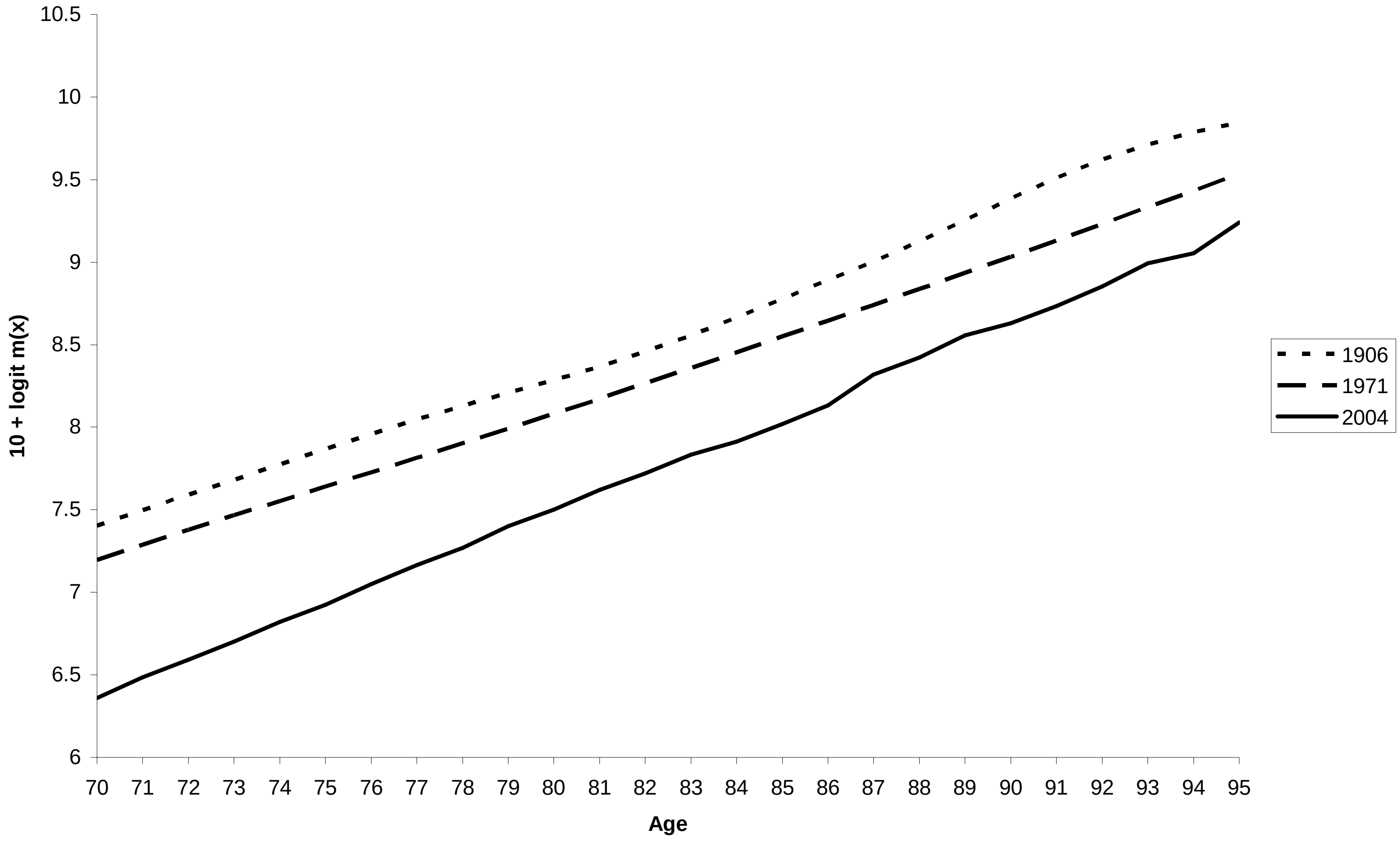

FIGURE 2: FEMALES IN ENGLAND AND WALES

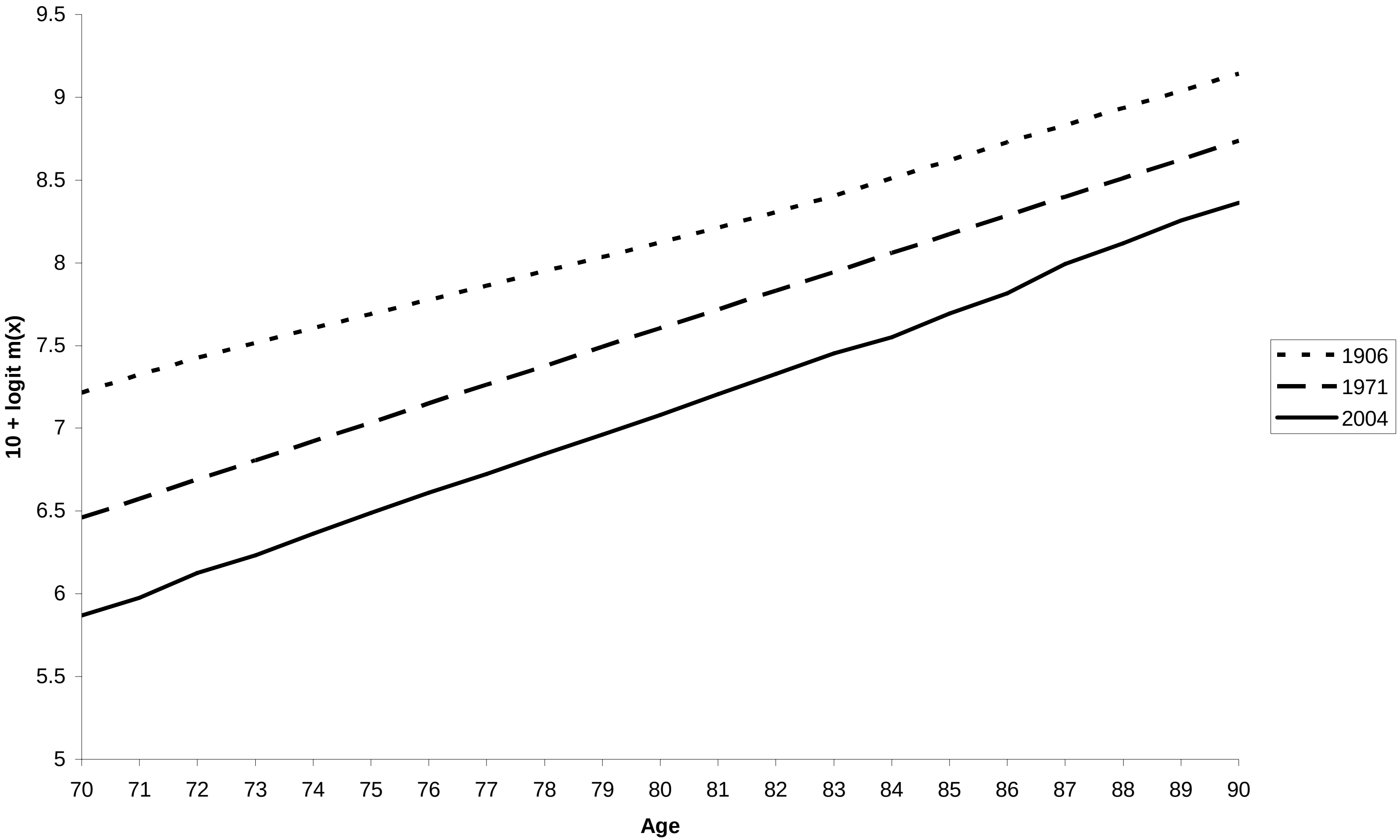

FIGURE 3: MODAL AGE OF DEATH - MALES IN ENGLAND AND WALES

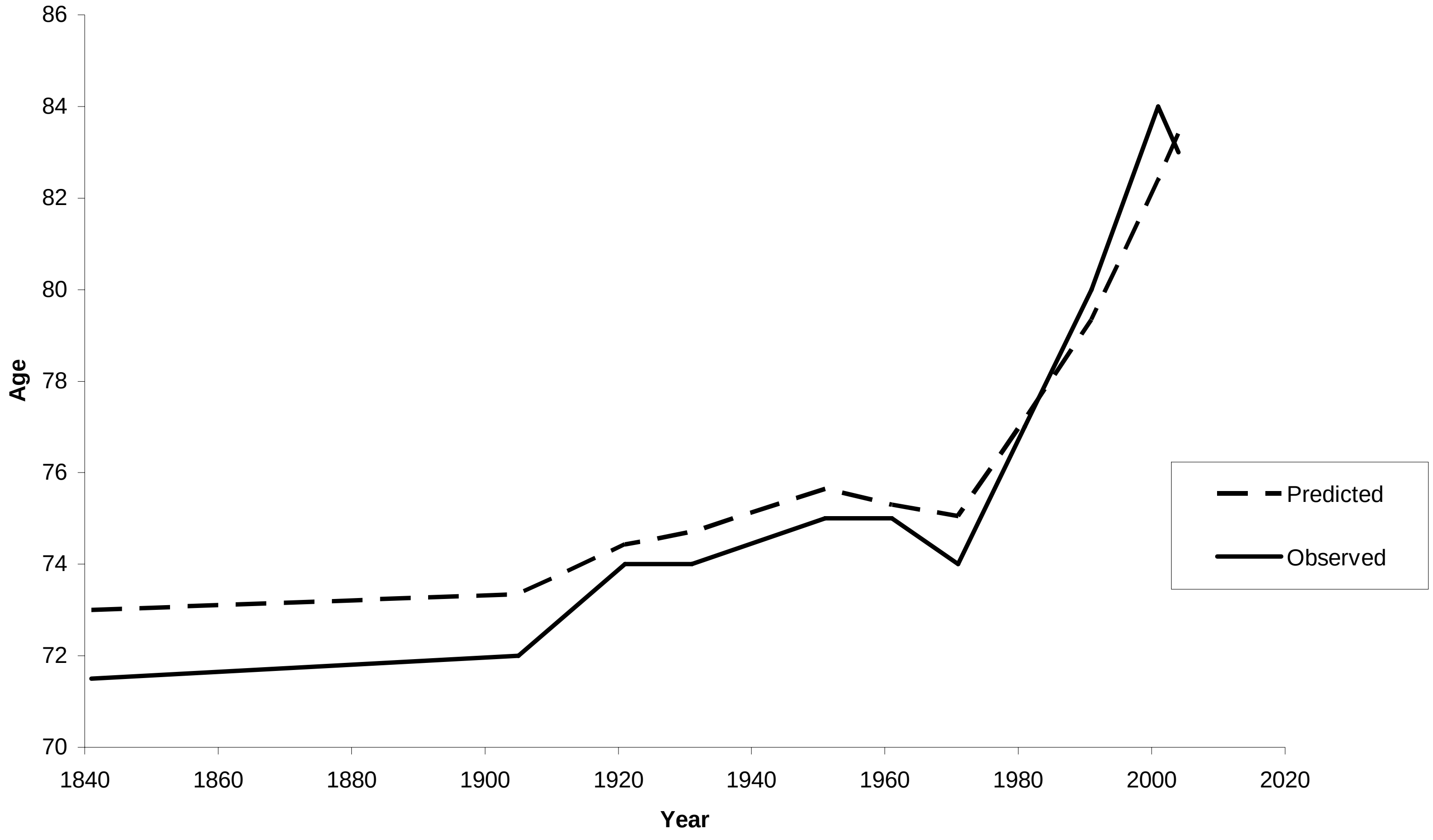

FIGURE 4 : MODAL AGE OF DEATH - FEMALES IN ENGLAND AND WALES

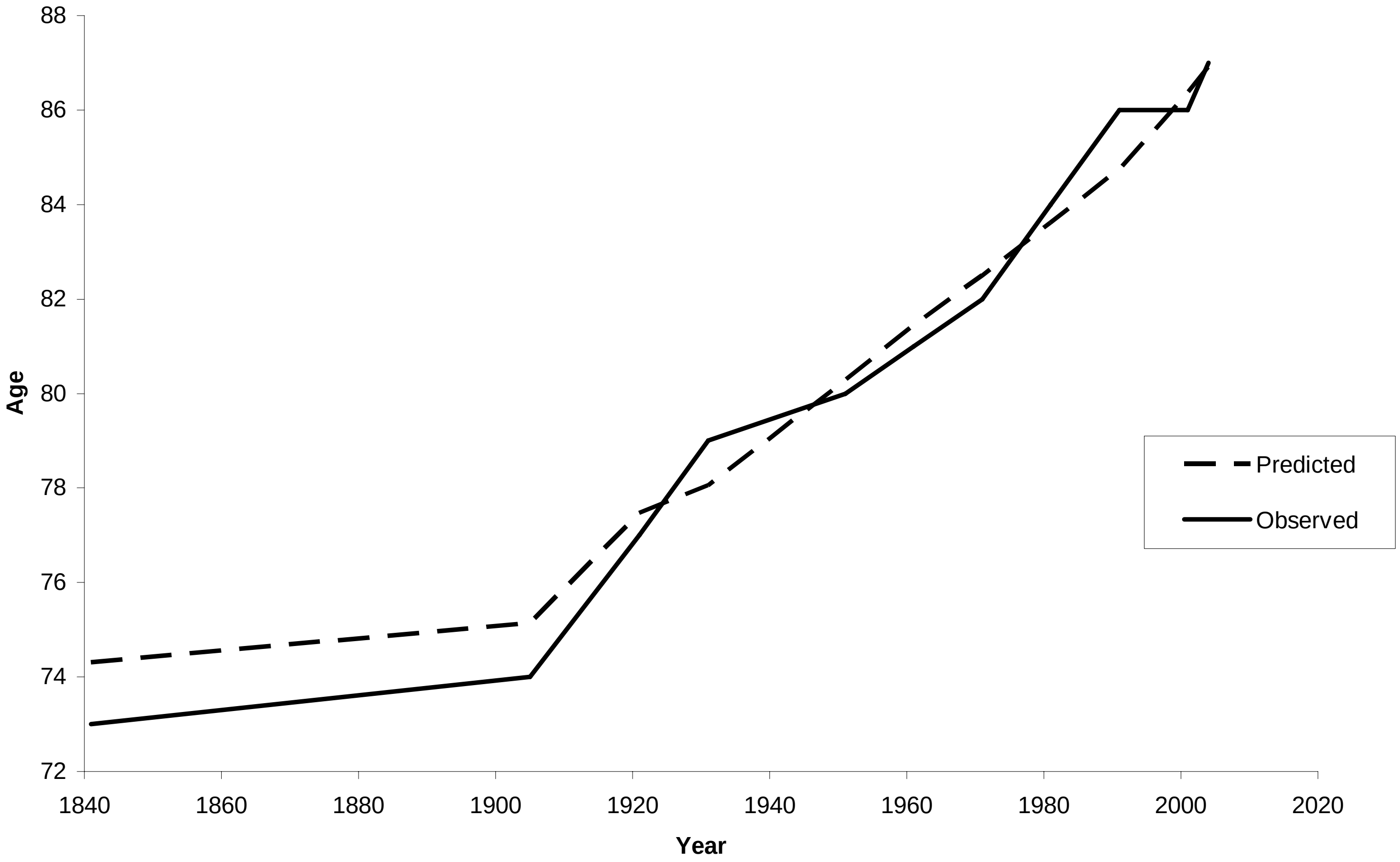

FIGURE 5: SCHEMATIC DIAGRAM