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Fertility, Human Capital, and Economic Growth over the Demographic Transition

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Introduction

The economic consequences of population age distributions feature prominently in discussions of the economic outlook in both low fertility countries where population aging is a concern, and in Third World countries where the demographic dividend may provide a boost to per capita income growth as fertility falls. On one level, macro economic consequences are calculated based on straightforward accounting: total dependency ratios or support ratios rise or fall, reflecting changing proportions of the total population in the conventional working ages. Such calculations reveal real and important consequences of demographic change but they are not the end of the story. On another level, a large literature explores other effects of these demographic changes on the capital intensity of economies, effects mediated by changing savings rates and labor force growth rates (Modigliani and Brumberg, 1954; Tobin, 1967; Mason, 1987; Higgins and Williamson, 1997; Kelley and Schmidt, 1995; Lee et al, 2000).

In the standard Solow neo-classical growth framework, low fertility leads to higher per capita consumption because slower labor force growth leads to capital deepening. This is the case if the saving rate is given (Solow 1956) or is golden-rule (Deardorff 1976). Samuelson raised the possibility, however, that in a model with age distribution and a retirement stage, over some relevant range lower population growth may reduce welfare because workers will have to support a larger number of elderly (Samuelson 1975; 1976). One purpose of this paper is to revisit Samuelson's conjecture, but with the emphasis on human rather than physical capital. Elsewhere we have argued that the response of life cycle saving when fertility and mortality are low will lead to an increased capital – labor ratio (a "second demographic dividend") which offsets the growing burden of old age dependency, provided that old age is not too generously supported through public or familial transfer programs (Mason and Lee 2006).

There has been less attention given to demographic consequences arising through effects on the side of human capital, although there have certainly been important contributions, mostly but not entirely theoretical (e.g. Becker, Murphy and Tamura, 1990, Mankiw, Romer and Weil, 1992; Jones, 2002; Montgomery, 2000). According to the quantity-quality theory of fertility, it is precisely because people want to spend more on each child that they reduce their fertility. To the extent that this increased spending takes the form of human capital investment, the adult labor productivity of these children will be higher, to some degree offsetting their reduced numbers.

Our analysis will use new cross-national estimates of human capital investment to quantify the quality-quantity tradeoff (Lee, Lee and Mason 2008; Mason, Lee, Tung et al. forthcoming). We hope to add time series estimates for a few countries in future work. While the foregoing discussion has emphasized implications for the consequences of population aging, implications for human capital investment during the early and mid demographic transition are equally important.

The first contribution of this paper will be to offer new empirical evidence about the quantity-quality tradeoff based on data from the National Transfer Accounts (NTA) project. It will present new estimates of public and private spending on education and health for children for a cross-section of countries, considering only expenditures and not time costs of education. It will answer the simple empirical question of whether lower fertility or youth dependency ratios at the national level are associated with higher human capital investment per child and whether this holds for both public and private sector investment in human capital. We are not able to draw any inferences about causality.

Based on these estimates and a simple model, we will then simulate the effects of changing fertility over the demographic transition on life time consumption. An initial model includes only workers and children, but our simulation model includes dependent elderly as well to incorporate concerns about population aging. We ask whether the increase in human capital associated with lower fertility might offset the greater cost of supporting the elderly in the older population that results from low fertility. Because there is considerable uncertainty in the literature about the effects of education on growth at the national level, we cannot come to any definite conclusion on this point.

Quality expenditures and human capital

In the literature on the quantity and quality of children (Becker and Lewis, 1974; Willis, 1972), all expenditures on children are combined and treated as investments in child quality. In a later literature all parental expenditures on children are viewed as raising future earning prospects for children which is the operational definition of quality (Becker and Barro, 1988). Our approach here differs from this tradition. We suggest that some expenditures on children have mainly consumption value for those children and yield vicarious consumption value for the parents, while others augment the children's human capital (HK). Specifically, we treat public and private expenditures on health care and on education as HK investment, and ignore all other kinds of expenditures on children.

The extended theoretical treatment of investments in child quality (e.g. Willis 1972) views quality as produced by inputs of time and market goods and services. It would certainly be desirable to include parental time inputs in the production of HK, but National Transfer Accounts, our data source, does not include time use so we are not able to do so. Furthermore, the literature on investment in education emphasizes the opportunity costs of the children who stay in school to receive further education, and often this is the only cost of education that is considered when private returns to schooling are estimated. These opportunity costs are certainly relevant, but for now we have included only direct costs in our measure.

Increased investment in HK can take place at the extensive margin by raising enrollment rates, which implies higher opportunity costs as in the traditional analysis. But it can also take place at the intensive margin through greater expenditures per year of education, for example through variations in class size, complementary equipment, hours of education per day, or teachers quality and pay rate. In East Asia much of the private spending appears to be of this sort, as children are sent to cram schools or tutors after the public school education is completed for the day. Such increased expenditures do not necessarily have an opportunity cost of the sort measured in traditional studies, and the increase in years of schooling would underestimate the increase in HK investment.

Estimates of Human Capital Spending in Relation to Fertility Across Nations

The National Transfer Accounts project provides the requisite data on age patterns of human capital investments per child and labor income for eighteen economies, rich and poor: the US, Japan, Taiwan, S. Korea, Thailand, Indonesia, India, Philippines, Chile, Mexico, Costa Rica, Uruguay, France, Sweden, Finland, Austria, Slovenia, and Hungary. Data are for various dates between 1994 and 2004. See Lee et al (2008) and Mason et al (in press). More detailed information is available at www.ntaccounts.org.

For each country, we have age specific data on public and private spending per child for education and health. We sum spending per child on education across ages 0 to 26, separately for public and private. We do similarly for health care, but this time limit the age range to 0-17. These are synthetic cohort estimates. We also have data on labor income by age[1] and we have calculated average values for ages 30-49, ages chosen to avoid effects of educational enrollment and early retirement on labor income. The data are averaged across all members of the population at each age, whether in the labor force or not, and include both males and females. They include fringe benefits and self employment income, and estimates for unpaid family labor. We express the HK (human capital) expenditures relative to the average labor income. These are our basic human capital data. For fertility we take the average Total Fertility Rate (TFR) for the most recent five year interval preceding the HK-NTA survey date, using United Nations quinquennial data.

Figure 1 plots the natural log of total HK expenditures (that is, public and private, health and education, summed over the ages indicated above) per child relative to labor income on the vertical axis, against the log of the Total Fertility Rate on the horizontal axis.

$$\ln(HK) = 1.924 - \underset{(8.04)}{1.1}*\ln(TFR), \qquad R^2 = .674$$

Figures 2 plots the private components of HK spending in relation to fertility, showing that the association is weak or nonexistent. Very low fertility countries include both East Asian ones with high private spending on education, and European ones with very low private spending on education, which accounts for the lack of a strong relationship.

Figure 3 shows a fairly strong association between public HK spending per child and TFR. This is mainly responsible for the strong relationship plotted in Figure 1. This relationship would not be revealed by analysis of individual level data within a country. At the aggregate level, this relationship might come about due to independent variations in support for public education, which in turn influence the relative prices of quantity and quality of children, and thereby influence fertility choices. Alternatively, the adult population in general (or median voter) may prefer a regime with lower fertility and higher HK investments per child. But in this case, why would not individual couples opt for higher fertility and higher investments, so long as these come through the public sector? In some cases, such as in East Asia, the reason may be that there are very high complementary private investments in child education, as are amply documented in NTA data. However, in Europe this cannot be the explanation because private expenditures are very low. In any case, the relationship does exist, and we will discuss some of its implications.

Figure 4 provides some evidence of substitution between public and private HK spending. We do not plot public HK spending against private, because a low fertility country might tend to have high values on both and conversely, which would obscure any possible substitution. For this reason the figure instead shows the proportional share of public and private spending out of the amount predicted by the regression reported above. Constructed in this way the two shares for a country do not need to sum to unity, nor in fact do they. The figure suggests strong substitution between public and private HK spending, with a slope of -.76, where a slope of -1.0 would indicate perfect substitution between the two.

The estimated elasticity close to -1 reported earlier suggests that total HK spending, given by TFR X HK spending per child, should be roughly constant as fertility varies. Figure 5 shows absence of any systematic variation of this measure with level of fertility. The next section explain how the pattern of variation observed in Figures 1 and 5 is related to the quantity-quality theory of fertility.

How the Empirical Pattern is Related to the Quantity-Quality Tradeoffs Model

In the basic quantity-quality tradeoff model of fertility choice, a couple has the utility function u(x,n,q) where x is parental consumption, n is number of children, and q is the quality of each of the identical and symmetrically treated n children. For our purposes we will view x as including all the ordinary consumption by the children as well as the parents, and reserve q=HK for the human capital investment per child. The parents' budget constraint is $Y=p_x x + p_q nq$, which is nonlinear in the numbers and quality of children.

In pedagogical presentations of the model (Becker, 1991, Chapter 5 or Razin and Sadka, 1995, Chapter 3) it is assumed for simplicity that the parents have already decided how to divide their income between own consumption and spending on children, so that the analysis focuses on the allocation of this chosen amount between numbers of children and spending on each, that is quantity and quality. It is also usually argued that the income elasticities of demand for both number and quality of children are positive if we hold shadow prices constant, but that the income elasticity is substantially larger for quality than for quantity. In this case, as income rises, the actual demand for number declines and the demand for quality rises more than proportionately with income, once shadow prices are allowed to vary. Here we make the same set of assumptions, but applied only to the HK expenditures on children, not on other aspects of children's consumption which are here grouped with parental consumption.

We will take literally the idea that the fraction of Y allocated to total HK investment in children, $p_q nq = Y_n$, with the fraction Y_n/Y allocated to total HK investment, can be taken as fixed across variations in income and fertility. This would result, for example, from an appropriately specified Cobb-Douglas utility function for HK vs non-HK expenditures. This is a strong assumption, but it appears to be roughly consistent with the empirical observations we report below.

The budget constraint for quantity-quality expenditures is:

$$Y_n = p_q n q$$

$$\frac{p_q q}{Y_n} = \frac{1}{n}$$

In our empirical exploration, we calculate the ratio of HK expenditure per child in money terms to the average labor income for age 30 to 49, call it w. A couple's life time labor income can be expressed as some multiple of this, say Y=80w (reflecting 40 years each of labor income for husband and wife) and Yn=dw. Then the budget constraint can be further rewritten as:

$$\frac{p_q q}{dw} = \frac{1}{n}$$

$$\ln\left(\frac{p_q q}{w}\right) = \ln d - \ln n$$

Because of the special assumptions made, the budget constraint after division by w is linear in the natural logarithms and has a slope of -1. This single line corresponds to the scatter plot in Figure 1. The constant term ln*d* is the log of the number of years of parental wages that are invested in HK. The income share of HK expenditures will be roughly given by d/80. In the regression reported earlier, the constant which estimates ln(*d*) is 1.9244, so d = 6.8, and the share of HK expenditures is roughly 8.5% or 1/12 (=6.8/80) of life time labor income.

The standard theory suggests that as income rises, fertility falls and investments in human capital rise, due to the interaction of quantity and quality in the budget constraint and the greater pure income elasticity of quality than of quantity. In Figure 6A, point A might correspond to a chosen allocation in a low income setting like the Philippines or India, with high fertility and low human capital investments per child. Point B might correspond to a high income setting like S. Korea or Spain with low fertility and high investments per child. In the standard diagram, we would see a series of budget constraints at varying differences from the origin, and a series of tangent indifference curves, generating the choices of fertility and p_qq. By dividing human capital expenditures by w, we collapse all the budget constraints onto the single line, so the explicit process of choice of A and B is invisible. (If Y_n were to vary with n, that would alter the slope of the budget constraint; if Y_n varies with Y, then division by w is not the appropriate transformation.)

To this point, we have only discussed the influence of income on the chosen outcome, but there are other factors at play that can lead to choice of a different point on the constraint line for a given level of income, like C, or to choices off of the average budget line, like D. Here are some of the other factors involved.

1) A number of factors influence the intercept ln(*d*): The lifetime years of work is high in low income countries and low in high income countries, so our assumption of 80 years of work by a couple will be systematically violated leading to differences in the intercept d across countries and income levels. Cultural differences between countries may lead to differences in the share of life time income devoted to human capital expenditures even holding income fixed. Differences in the relative price of parental consumption, p_x and human capital, p_q, could also lead to choice of different shares of life time income devoted to human capital expenditure as well. The changing availability of new consumer goods could also affect the relative preference for parental consumption versus expenditures on children.
2) We have used fertility (TFR) as our measure of quantity, but more properly it would be surviving children, and survival probabilities vary from country to country.
3) The relative preference for human capital per child over numbers of children might be influenced by differences in the rate of return to education or by older age survival probabilities, as has often been suggested. Cultural and religious differences might also lead to different choices, given the same level of income.
4) The model should be expanded to include a fixed price of number of children, p_n, not shown in the equations above (see Becker, 1991). Examples are financial incentives or disincentives for child bearing such as family allowances in Europe or the fines of the one child policy in China. The availability of contraceptives effectively raises the price per child, because having a child no longer permits you to have sex that would previously have been risky.

For all these reasons and more, we can meaningfully consider the effects of an exogenous change in fertility on human capital investment, and the effects of an exogenous change in human capital investment on fertility. Furthermore, the list of factors suggests some possible instruments for identifying these effects, that could be useful in future research.

In the simulations to which we turn later, we will take the time path of fertility as exogenously determined, and attempt to trace out its implications for human capital investment and income growth. A different approach, not taken here, would be to incorporate explicitly the effect of rising income on fertility and human capital investments drawing on the quantity quality theory. In the resulting simulations, income, fertility and human capital would all change endogenously.

Education and Economic Growth in Recent Theoretical and Empirical Literature

In a prominent article, Becker, Murphy and Tamura (1990) assign a central role to human capital as the main driver of economic growth, with output of consumption goods proportional to the stock of human capital (constant returns), and human capital per child proportional to the human capital of the parent generation. If it escapes a Malthusian trap, then the system converges to a steady state growth path with constant fertility, growing human capital per person, and a growing rate of return to human capital. In models of this sort human capital obviously has a very important role and declining fertility could apparently lead to faster aggregate economic growth.

In the endogenous growth model of Jones (2002), some returns to education are captured by the national economy, but the biggest payoff is global and shared, with population growth raising the numbers of educated people participating in research and development, which drives global technological progress. Fertility reduction in one country would permit greater investments in HK per child and higher per capita income, and the country could continue to benefit from new ideas generated abroad. A global downturn in population growth would probably reduce per capita income growth in this model, however, although it is hard to be sure, since Jones does not link population growth and per capita investments in education.

We will now briefly discuss some of the literature on the economic effects of investment in human capital, HK. This is a deep and difficult topic, and we are looking for some simple assumptions and quantifications that might be consistent with a portion of the literature. The literature is not internally consistent, in any case.

Most of the literature estimates private rates of return to education based primarily on the opportunity cost of the time of the student who invests in an incremental year of education, although sometimes tuition costs are also included. Card (1999) provides a recent analytic overview of this literature and reviews many IV studies, finding that in general the IV studies report even higher rates of return to education than do the OLS studies, with a broad range centered on about 8% per year. Heckman et al (2008) estimates rates of return for the US based on extended Mincer-type regressions allowing for various complications, and also including tuition, but without IV to deal with the endogeneity of schooling. They report rates of return in the range 10 to 15% or higher for the contemporary US (for a college degree, given that one already has a high school degree).

For our purposes this literature has two main problems: it focuses exclusively on the extensive margin of years of schooling (as opposed to increased investment in a given level of education) and it focuses exclusively on private rates of return rather than including social rates of return, which could be higher (due to externalities) or lower (due to inclusion of direct costs).

There is also a considerable literature that assesses the effect of education on per capita income or income growth rates. These estimates should reflect both full costs of education and spillover effects. One approach treats HK in a way similar to K, as a factor of production for which an output elasticity can be measured. Studies taking this approach sometimes report similar estimated elasticities of output with respect to labor, HK, and K (e.g. Mankiw, Romer and Weil, 1992; Lau, 1996). In another approach, HK is viewed as raising the rate at which technological change can be adopted and therefore HK is expected to affect the growth rate of output rather than its level (Nelson and Phelps, 1966).

The earning functions fit on individual data are generally specified in semi-logarithmic form, which suggests that the underlying function linking the wage w to years of schooling has the form: $w = e^{\psi E}$ where ψ is the rate of return to years of education E. This suggests that human capital H or HK (two notations for the same variable, unfortunately) in relation to schooling level also has this form. Cross-national estimates of aggregate production functions including human capital as an input, from this perspective, should look like $Y = AK^{\alpha}(HL)^{1-\alpha} = AK^{\alpha}(e^{\psi E}L)^{1-\alpha}$, where L is the labor force and HL is therefore the total amount of human capital given (Jones, 2002).

However, this is not the form that these cross-national regressions take. Instead, variables like median years of schooling completed or proportions enrolled in secondary education are used to measure H (e.g. Mankiw et al or Barro and Sala-i-Martin, 2004:524). The difference is important. Under the exponential version, the human capital increment associated with the 15th year of schooling is four or five times larger than that associated with the first year of schooling, when ψ =.1. Appendix 1 compares the direct estimates of aggregate level effects with the implications of the individual level ones. We conclude there that it is reasonable to take as our baseline an assumption an elasticity for .33 for output with respect to human capital.

In Jones (2002) there is a second fundamentally important effect of education, which is to raise the world stock of ideas which is the basic force driving economic progress. But Jones assumes that these ideas are immediately available to every country in the world, so investing more in the HK of one country will have very little effect on its own factor productivity A. Therefore we will ignore that component for present purposes, while remaining in principle consistent with at least this particular version of endogenous growth theory.

Our measure of human capital investment is quite different than E, some measure of educational attainment, as discussed earlier. Because we are not aware of any estimates of the returns to human capital as we measure it, for present purposes we will model the gains to human capital as we measure it as if it were simply E.

A Simple Model of Fertility, HK investment, and Economic Growth

Here we develop a simple model of a population with two age groups, children and workers, which permits us to focus on the human capital and wage dynamics. Later, when we simulate the behavior of the economy over time, we will add a third age group, the retired elderly, which will allow us to balance the advantages and disadvantages of low fertility.

Notation

H=human capital investment per child, subscripted by their generation.
F=fertility per generation, subscripted by parental generation.
W=wage of working generation (parental).
N_x=size of generation x. x=0,1.
T=total wage bill indexed on generation of current workers.

Assumptions, Equations and accounting identities

We will use a simple model in which the births of one period are the workers of the next period, then they die (we do not distinguish between the working age population, N1, and the labor force, L). F is the net reproduction rate, the number of female births surviving to adulthood per mother, which we will henceforth refer to as fertility.

$N1_{t+1} = F_t N1_t$.

Here time is measured in generations.

Investment in human capital relative to parental generation wages is a function of the level of parental fertility:

$H_{t+1}/W_t = h(F_t)$ = expenditure on human capital H relative to wages W by parental generation t per child in generation t+1. Thus when the parental generation has more education, and hence a higher wage, their children will receive greater HK investment at any level of fertility.

$W_t = g(H_t)$ is wage as a function of amount of HK investment a generation received.

Therefore:

$$H_{t+1} = h(F_t)W_t = h(F_t)g(H_t)$$
$$W_{t+1} = g(H_{t+1})$$

Note that these equations introduce a lag of one generation between investment in human capital of a generation of children and its effect on their labor productivity when they enter the labor force.

From this it follows that the total wage bill in the two periods is:

$$T_t = N1_t W_t$$
$$T_{t+1} = N1_{t+1} W_{t+1}$$
$$T_{t+1} = F_t N1_t g\left[h(F_t)W_t\right]$$

The growth rate of the wage bill is:

$$T_{t+1}/T_t = F_t N1_t g\left[h(F_t)W_t\right]/N1_t W_t$$

Which simplifies to:

(0.1) $$T_{t+1}/T_t = F_t g\left[h(F_t)W_t\right]/W_t$$

Special Case

Consider the special case in which g and h are constant elasticity functions, as follows:

$h(F_t) = \alpha F_t^{\beta}$

$g(H_{t+1}) = \gamma H_{t+1}^{\delta}$

Then we have:

(0.2) $T_{t+1}/T_t = \alpha^{\delta}\gamma W_t^{\delta-1}F_t^{1+\beta\delta}$
Or
$T_{t+1} = \alpha^{\delta}\gamma W_t^{\delta-1}F_t^{1+\beta\delta}T_t$

Note, however, that since the growth rate of T is declining with number of generations of time, it may pass through zero on its way to a lower asymptote (as is shown in the simulations) so we should instead look for the steady rate of growth of T, rather than a steady state level. Using (0.2) to find the growth rates at time t and t+1 and equating them we get:

(0.3)
$$W_t^{\delta-1}F_t^{1+\beta\delta} = W_{t+1}^{\delta-1}F_{t+1}^{1+\beta\delta}$$
$$\left(W_{t+1}/W_t\right)^{\delta-1}\left(F_{t+1}/F_t\right)^{1+\beta\delta} = 1$$

This says that the exponentiated product of the growth factors for wages and fertility must be unity, or if re-expressed in term of continuous growth rates, would say that the weighted sum of the growth rates of wages and fertility must be zero. If we assume that fertility is stationary, then this tells us that the wage rate must likewise be stationary.

Discussion

From equation (0.2) we can derive additional results.

1) Lower fertility will raise the rate of growth of the wage bill (and presumably GDP) if $1+\beta\delta<0$. Since $\beta<0$ (higher fertility leads to lower *HK* investments), while $\delta>0$ (higher human capital leads to higher wages), the real question is whether the absolute value of $\beta\delta>1$.

2) We also see that a higher value of parental wages, W_t, leads to a lower rate of growth provided that $\delta<1$ which is likely to be satisfied. This reflects diminishing returns to human capital, so that the greater the human capital of the parental generation, the smaller the wage gain from investing more in their children's human capital.

3) For a given and constant level of fertility, total wages T=N1W will approach an asymptotic growth rate.

But note that in general the population will be increasing or decreasing at this point, since the level of fertility is arbitrarily set and constant, so the wage rate *W* must be decreasing or increasing at exactly the negative of this rate when *T* is at this asymptotic value. With low fertility as in, say Korea, there would be perpetually rising wages at the steady state rate *1/F*, so with F close to .5 as it is now in Korea, wages would be doubling every generation. In the long run, the rate of return to transfers in a PAYGO system will be 0 according to this little model, whether fertility is high or low.

4) Given the result above, we will want to analyze the relation between fertility and wage growth. From the equations above, we can derive the general result that:

$$W_{t+1} = g\left[h\left(F_t\right)W_t\right]$$

In the special case this becomes:

$$W_{t+1} = \gamma\left[\alpha F_t^{\beta} W_t\right]^{\delta}$$
$$W_{t+1} = \left(\alpha^{\delta}\gamma\right) F_t^{\beta\delta} W_t^{\delta}$$
$$W_{t+1}/W_t = \left(\alpha^{\delta}\gamma\right) F_t^{\beta\delta} W_t^{\delta-1}$$

Noting that $\beta\delta<0$, we have the plausible result that for a given level of parental human capital and wages, lower fertility leads to higher wages in the next generation. Closely related to this result, we see that lower fertility leads to higher wage rate growth from generation to generation.

We can also find the value of W for which $W_{t+1}=W_t$:

$$\left(\frac{1}{\alpha^{\delta}\gamma}\right)^{\frac{1}{\delta-1}} F_t^{\beta\delta/(1-\delta)} = \hat{W}$$

Since $\beta\delta<0$, this expression tells us that higher fertility is associated with lower wages in steady state, with an elasticity of roughly -.5.

In a later section we will supplement these steady state comparative static results with dynamic simulations over the course of a stylized demographic transition. Before doing so, however, we will pause to discuss the empirical association we find between national human capital investment per child and the level of fertility.

Simulation Analysis

The simulation analysis implements the simple theoretical model discussed in the previous sections. Some modest elaborations have been introduced in order to achieve additional realism and to allow a more complete assessment of the implications. The basic mechanisms through which fertility decline, human capital, and economic growth interact are unaffected by the elaborations. Variables are defined in Table 1.

<Table 1 about here>

The population consists of three age groups: N0(t) are children, N1(t) are workers, and N2(t) are older adults, who do not work, in year t. The total population, N(t), is the sum of these three age groups. The fertility rate (or net reproduction rate) is F(t). The proportion of workers surviving to old age is s(t). The population is closed to immigration. Hence:

N0(t) = N1(t) F(t)
N2(t) = s(t)N1(t)/F(t-1)

The wage is determined by the worker's human capital, H(t), using a constant elasticity production function, i.e.,

$$W_t = \gamma H_t^{\delta}$$

The human capital of workers in year t is equal to the investment in human capital in period t-1 which, in turn, is a constant elasticity function of the fertility rate:

$$H_t = i_{t-1}W_{t-1} = \alpha F_{t-1}^{\beta} W_{t-1}$$

Substituting, the wage of workers is given by:

$$W_t = \gamma\alpha F_t^{\beta\delta} W_{t-1}{}^{\beta}$$

There is no capital in this simple economy and, hence, total income is equal to total wages, i.e.,

$$T_t = \gamma\alpha F_t^{\beta\delta} W_{t-1}{}^{\beta} N1_t$$

All output is consumed or invested in human capital. The consumption of children relative to workers is a_0 and the consumption of the old relative to workers is a_2. Hence, total consumption is equal to:

$$C_t = T_t - i_t N0_t$$

Consumption per equivalent adult is:

$$c_t = C_t / (a_0 N0_t + N1_t + a_2 N2_t)$$

Although transfers are not emphasized in this paper, note that children and the elderly are supported entirely through transfers from workers. This follows from an important feature of the model – that the only asset is human capital. The parameters, their values, and sources are provided in Table 2.

<Table 2 about here>

Demographic variables, fertility and adult survival, are exogenous. The baseline simulation analyzes the transition in F from a peak value of 2.0. Replacement fertility, F=1, is reached after one period. Fertility continues to decline for two periods reaching a minimum of 0.6. Thereafter, fertility gradually recovers eventually reaching replacement level. The baseline simulation also incorporates a rapid transition in adult mortality with the proportion surviving to old age rising from 0.3 to 0.8 over the course of the demographic transition.

The model is initialized by assuming that a pre-transition steady-state existed in t = -2. The NRR increased from 1.2 in t = -2 at a constant rate to reach 2 in t =0. Adult survival is held constant during this period. The age structure in t = 0 reflects these early demographic changes.

The key demographic variables are presented in Table 3.

<Table 3 about here>

The simulation covers seven periods (generations) or roughly two centuries during which there are three distinct phases.

Boom: Temporarily high net fertility which leads to an increase in the share of the population in the working ages as measured either by the percentage of the population who are workers or the support ratio.[1] The boom lasts for a single generation of thirty years.[2]

Decline: Declining fertility is leading to a decline in the share of the working age population and the support ratio. In the simulation this lasts for two generations or approximately 60 years.

Recovery: The share of the working age population and the support ratio rise as a consequence rising fertility with a one generation lag. In the baseline simulation, recovery last for two generations or approximately 60 years.

For the final two periods of the simulation, net fertility is held constant at the replacement rate.

Note that the timing of fertility decline and recovery are not based on any particular historical experience. A number of countries have reached very low fertility rates similar to those in the baseline simulation, but it is unknown when they might recover. Japan has had a TFR of 1.5 or less for almost two decades at this point.

Table 4 reports human capital variables for the baseline simulation. The share of the wage or labor income invested in the human capital of each child is reported in the first column. Human capital spending per child is low in period 0 because there are so many children relative to the number of workers. The investment in human capital in children in period 0 is actually less than the human capital of the current generation of workers who were members of a smaller cohort. The large cohort enters the workforce in period 1 leading to the first demographic dividend. Note that the average wage has declined from period 0 to 1 because members of the large cohort have less human capital than the previous generation of workers. During the first dividend period, then, the favorable impact of the entry of a large cohort of workers is moderated because the large cohort is disadvantaged with respect to its human capital.

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[1] The support ratio is calculated as the number of workers adjusted for age variation in productivity divided by the number of consumer adjusted for age variation in consumption "needs".

[2] Using more detailed age data, estimates of the first dividend stage are typically between one and two generations long. For East and Southeast Asia, a region with rapid fertility decline, Mason estimates the first dividend period lasts 46 years on average.

The impact of low fertility on human capital occurs during the fertility decline phase. Human capital spending per child increases from 4.7 percent of the average adults wage in period 0 to 10.0 percent in period 1 to 17.5 percent in period 2. With a one generation lag this leads to greater human capital and a higher wage. The peak in human capital investment per child is reached in period 2 and the peak in human capital is reached in period 3.

Note that the trend in human capital investment depends both on the share of the wage invested in human capital per child but also the wage. Thus, human capital has a multiple effect. The wage or the human capital of the current generation of workers depends on the human capital investment they received and also the human capital investment received by their parents' generation.

During the recovery period fertility is rising and, hence, human capital investment is declining. With a lag the human capital of the workforce declines as does the average wage until an equilibrium is reached at replacement fertility.

<Table 4 about here>

Key macroeconomic results are reported in Figure 6. The support ratio is of interest because it marks the three demographic phases of interest and also because it tells us how consumption and income would vary in the absence of investment, human capital or otherwise. If all labor income is consumed and none invested, consumption per equivalent adult exactly tracks the support ratio. Following the boom period labor income would increase by about 20 percent. Thereafter, foregoing the second dividend, fertility decline would have a severe effect leading to a decline in consumption by one-third. As fertility recovers and the working population rises relative to the older population, consumption would recover but only to about 5 percent below the pre-transition level. Thus, the first dividend would not only be entirely transitory but very low fertility would have a strongly adverse effect on standards of living with a one generation lag.

<Figure 6 about here>

With human capital investment the outcome is very different. GNP per capita grows about as rapidly than the support ratio during the first dividend period. Further gains are realized as the impact of human capital investment is felt. At the peak GNP per capita is 50 percent above the pre-transition level. Per capita GNP declines as fertility increases and spending on human capital declines, but per capita GNP stabilizes at a level about forty percent above the pre-transition level.

Consumption per equivalent adult rises much more slowly than per capita GNP or the support ratio during the boom period. The reason for this is two-fold. First, the share of GNP devoted to human capital increases moderately so less is available for consumption. Second, the decline in the relative number of children has a larger impact on per capita GNP (children count as 1) than on C per equivalent adult (children count as 0.5).

Thereafter consumption per equivalent adult rises markedly achieving a 20 percent increase as compared with period 0. Consumption stabilizes at a higher level – between 15 and 20% above the pre-dividend level.

They key feature of this simulation is that human capital investment has allowed the first dividend to be converted into a second dividend. The affects of population aging are reversed as large cohorts of less productive members are replaced with small cohorts of more productive members.

Variations in parameters and demographics

How sensitive are the results to variations in parameter values and demographic variables? We consider variations in the elasticity of human capital spending with respect to fertility and the elasticity of wage with respect to human capital. We also consider how variations in the fertility transition influence the outcome. In all cases we compare only consumption per equivalent adult as the preferred measure of the standard of living.

Variation in the elasticity of human capital investment with respect to fertility given all other baseline variables is considered in Figure 7. During the boom period a higher elasticity translates no consumption boom because the high fertility cohort has substantially lower human capital than the preceding generation. Moreover, in period 1 when fertility is at replacement level spending on human capital is much higher given the high elasticity case. Hence, less is available for consumption. Thereafter the gains from investment in human capital become more apparent. The gaps between the low and the high elasticity range from about 10 to 20 percent.

<Figure 7 about here>

In Figure 8 we consider the implications of variation in the elasticity of the wage respect to human capital. As compared with the baseline value of 0.33, we present simulations for an elasticity of 0.16 and an elasticity of 0.5. In a qualitative way varying the effect of human capital on the wage has a similar effect to varying the effect of fertility on human capital. If human capital has a small effect on the wage, human capital investment will moderate the effects of variation in age structure on consumption per equivalent adult to a lesser degree. If human capital has a large effect on the wage, then human capital investment can lead to a very substantial second dividend and overcome the adverse effects of low fertility and population aging.

<Figure 8 about here>

Finally we consider cases when both elasticities vary. The comparison in Figure 9 shows the extreme cases when both elasticities are large or both are small. Perhaps the only feature of this that is interesting is that in the high human capital investment case consumption actually declines significantly as the baby boom cohort reaches the working ages. The gain from a larger cohort is much smaller because the cohort is relatively uneducated. Moreover, workers in period 1 are spending a large amount on human capital.

<Figure 9 about here>

The final set of simulations explores how features of the fertility transition influence the path of consumption given the baseline parameters values (Figure 10). Three scenarios are considered. In the first, the fertility rate declines slowly, over two generations rather than one, to replacement level and declines no further. In the second scenario fertility declines rapidly, over one generation, to replacement fertility and declines no further. In the third scenario, fertility declines slowly to sub-replacement level, 0.6 as in the baseline scenario, and recovers at a speed similar to that in the baseline. Note that in all cases the demography at the end of the simulation is identical. Hence, steady-state consumption per equivalent adult will be the same at the end of the simulations. Our interest here is in the paths to that steady-state. In the simulation results presented here steady-state has not yet been entirely realized. By period 9 (not charted) steady-state has been reached with consumption per equivalent adult 16 percent higher than in period 0.

Perhaps the most striking difference in the simulations is that the slow fertility transition to replacement fertility, given the baseline parameter values, results in a consumption path that declines when the first cohort of baby boomers enters the workforce and increases only begins to increase when the second wave of baby-boomers enters the workforce in period 2. In this scenario the rise in the old age population never is sufficient to depress consumption per equivalent adult. In the other three scenarios, consumption declines in one period because of the increase in the share of the population at older ages.

<Figure 10 about here>

Conclusion

A number of potentially important issues related to changes in population age structure are explored in this paper, albeit in a very preliminary way. The key idea is that it is insufficient to focus on the relative number of people in age groups. The productivity of those individuals also matters. Because investment in human capital and fertility are closed connected, the total amount produced by a cohort will not decline in proportion to its numbers. Indeed, it is possible that it could rise as cohort size falls.

In the context of the demographic transition the potential tradeoff between productivity and numbers raises interesting questions. First, does the first dividend have a diminished effect on per capita income because the large entering cohorts of workers will have lower human capital per capita than preceding cohorts? Second, is investment in human capital a mechanism by which the first dividend can be invested in future generations – generating a lasting second dividend? The third question concerns Samuelson’s conjecture. Does lower fertility and slower population growth always lead to higher standards of living or can fertility be too low (aging too great)?

The implication of rising fertility for human capital investment and economic growth is relevant at two points over the demographic transition as modeled in this paper. Before childbearing begins to decline the net reproduction rate increases due to reduced infant and child mortality. Also during the recovery period the rise in fertility leads to a decline in human capital investment. In both cases rising fertility leads to an increase in the share of the working population and a demographic dividend, but one that will be more modest if the larger generation of workers is less productive than the preceding one. This is an interesting possibility but the evidentiary base is weak. The data used to estimate the tradeoff between fertility and human capital investment come from countries that differ in the extent to which their fertility rates have declined, but no country is represented prior to the onset of fertility decline or at early stages of the decline. The existence and magnitude of the quantity-quality tradeoff may be very different during other phases of the demographic transition and dividend.

Our empirical results suggest that human capital expenditures per child are substantially higher where fertility is lower, to the extent that the product of the Total Fertility Rate and human capital spending per child is roughly a constant share of labor income across countries, although total spending per child falls with fertility. About one twelfth of parental life time labor income is spent on human capital investments, in both rich Korea with a TFR near one, and in the low income Philippines with a TFR near three. This suggests that during the demographic transition, a portion of the first demographic dividend is invested in human capital, reinforcing the economic benefits of fertility decline. It also suggests that the very low fertility in some countries like Austria, Japan or S. Korea is associated with an increased human capital investment per child that might reduce or at least postpone the support problems brought on by population aging.

Second, human capital investment is a potentially important mechanism by which a second demographic dividend can be generated. Fertility decline leads to substantial population aging and a rising dependency burden. As measured by the support ratio, the dependency burden can be as great or greater at the end of the transition as at the beginning. Although we have not emphasized this feature of the simulation model, the transfers from workers to the elderly are very substantial at the end of the transition. Standards of living as measured by consumption per equivalent adult can be sustained at relatively high levels, however, if the quantity-quality tradeoff is sufficiently strong and if human capital has a sufficiently strong effect on productivity. If the rate of growth is raised sufficiently by human capital investments, then even the share of output transferred to the elderly need not rise much.

The third issue is whether slower population growth is always better. This question can be answered using simulation results not reported in the main body of the paper. We allowed the elasticity of human capital with respect to fertility to vary as in the sensitivity analysis reported above. Steady-state consumption per equivalent adult was calculated using NRRs of 1.2, 1, 0.8, and 0.6. If the elasticity of output with respect to human capital is set to the baseline value of 0.33, slower population growth leads to higher consumption per equivalent adult for any of the elasticities used to measure the quantity-quality tradeoff. If the elasticity of output with respect to human capital is set to 0.16

(well below the level implied by rate of return estimates as discussed earlier), and if the elasticity of human capital with respect to fertility is set to -0.7 rather than -1.0, however, consumption per equivalent adult is higher for an F of 1 than for an F of 0.8 or 1.2.

There are many important qualifications that should be kept in mind in considering these results. First, the model of the economy is highly stylized in several important respects. We do not allow for capital, although this is an issue that we have explored rather extensively elsewhere. There is no technological innovation, although we believe this can be introduced with little effect on the conclusions. By using only three age groups we are relying on a very unrealistic characterization of the population and the economy. A model with much greater detail would be better suited to providing a quantitative assessment of the issues being explored here, and we believe we can construct one from the building blocks introduced here.

Second, the role of human capital in economic growth is unsettled in the literature. Estimates of the importance of human capital vary widely. It is very likely that the effect of human capital varies across countries depending on a host of factors that are not explored here. At this point we can do no better than allow for a wide range of possible effects.

Third, the empirical basis for quantifying the quantity-quality tradeoff is also weak, although it is widely accepted that such a tradeoff exists. An interesting result here is that the tradeoff is a feature of public spending rather than private spending. Caution should be exercised in interpreting the results presented here because we are not asserting any particular causal relationship between fertility and human capital. Thus it would be quite inappropriate to argue for fertility policy of any sort based on the simple cross-sectional relationship between human capital spending and fertility. We are only saying that countries with lower fertility are spending more on human capital per child. Because this is so, low fertility and population aging may not have the adverse affects on standards of living that are widely anticipated. This conclusion holds even though the elderly rely entirely on transfers from workers for their material support.

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Appendix 1. Comparison of Estimates for Earnings Functions and Aggregate Production Functions

We will compare steady states with different levels of education without worrying about the timing of the income increase relative to the increase in education, although the lag between the two is very substantial. In order to compare the estimates in the two literatures, we will calculate the implied derivatives in the neighborhood of S=10. For implementation of our simulation model we will use a specification similar to the aggregate cross-national one for simplicity.

We take the literature on individual rates of return as a point of departure, using 10% as a typical estimated value. As discussed above, following Jones (2002) we have:

(0.4)
$$Y = AK^{\alpha}\left(e^{\psi E}L\right)^{1-\alpha}.$$

We take A as fixed. H is produced via the production function shown, where E is the number of years of education and also the reduction in labor time spent in acquiring it. A includes an appropriate scaling factor to adjust the units of measure of H.

Acquiring education E entails an opportunity cost in the form of reduced labor. Suppose that a person who gets 0 years of education would work from age 10 to 65, or for 55 years, and that this amount is reduced by a year for each year of education. Thus workers with 10 years of education would work for 45 years and those with 15 years would work 40, for example.

(0.5)
$$L = (55 - E)N$$

where N is the number of working age people. This simple expression does not incorporate variation in labor productivity by age and, thus, will overstate the opportunity cost of primary school education but seems reasonable thereafter. We will assume that our empirical HK measure, expenditures on HK including health, equals E, with the parameter A in (0.4) absorbing an appropriate scaling factor.

We seek the effect of a change in E on Y, total output, in proportional terms.

(0.6)
$$\frac{dY/Y}{dE} = \left(1-\alpha\right)\frac{dL}{dE}\Big/L + \left(1-\alpha\right)\frac{dh}{dE}\Big/h$$
$$\frac{dL/L}{dE} = \frac{-1}{55-E}$$
$$\frac{dh/h}{dE} = \left(1-\alpha\right)\psi$$

Substituting and simplifying we find:

(0.7)
$$\frac{dY/Y}{dE} = \left(1-\alpha\right)\left(\psi - \frac{1}{55-E}\right)$$

The term (1-α)/(55-E) is the opportunity cost to the economy due to the loss of workers who are staying in school. The term $(1-\alpha)\psi$ is the gain due to the increased productivity of having more educated workers.

Evaluating this at E=10 years of education, with α = .3 and ψ =.1 (the return to an 11th year of education), then the net effect of one more year of education is to raise total output by 5.6%. With ψ =.07 it would be 3.5%, and with ψ =.14 (closer to the estimates of Heckman et al, 2008) it would be 8.4%.

Now compare this result to the Mankiw et al (1992) or Lau (1996) result, which we take to be equal coefficients for capital, human capital, and raw labor. Based on this specification, we find:

(0.8) $$\frac{dY/Y}{dE}=\frac{1}{3E}$$

With E=10, this is .033, which is reasonably close to the .056 or .035 we derived above, but rather different than the .084.

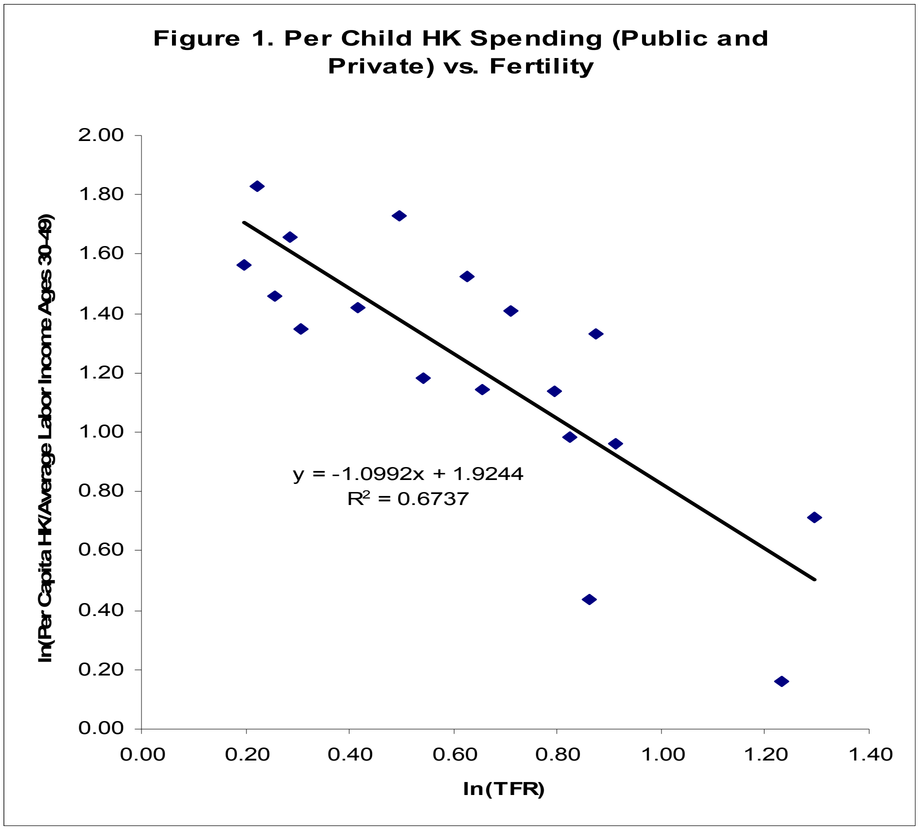


Figure 1. Per Child HK Spending (Public and Private) vs. Fertility
ln(Per Capita HK/Average Labor Income Ages 30-49)
2.00
1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
y = -1.0992x + 1.9244
R^2 = 0.6737
0.00
0.20
0.40
0.60
0.80
1.00
1.20
1.40
ln(TFR)

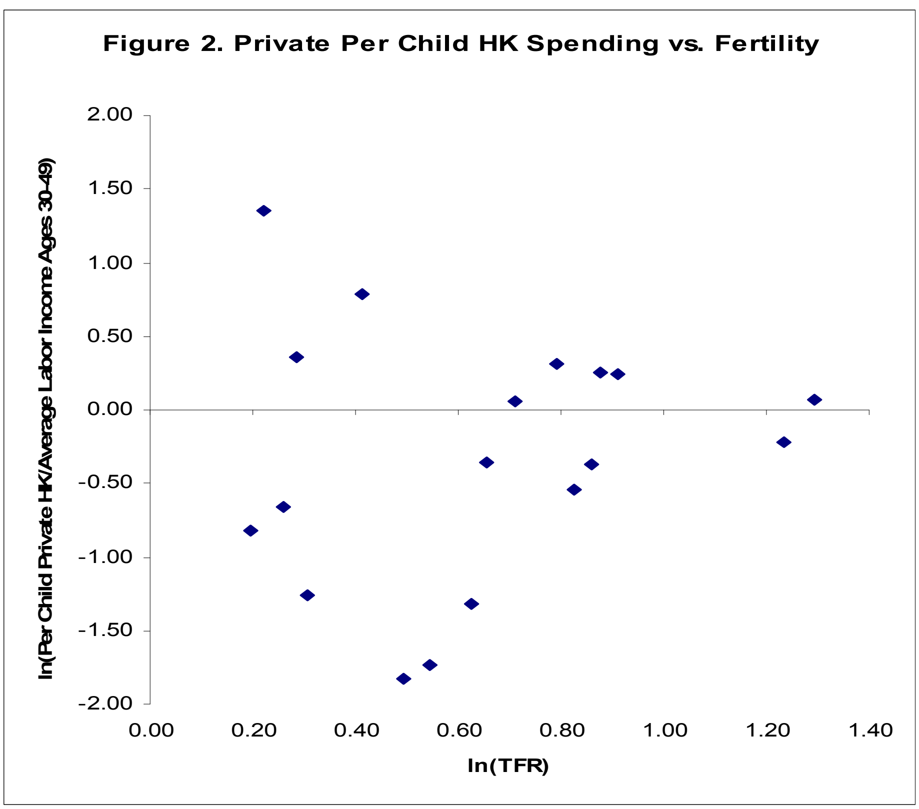
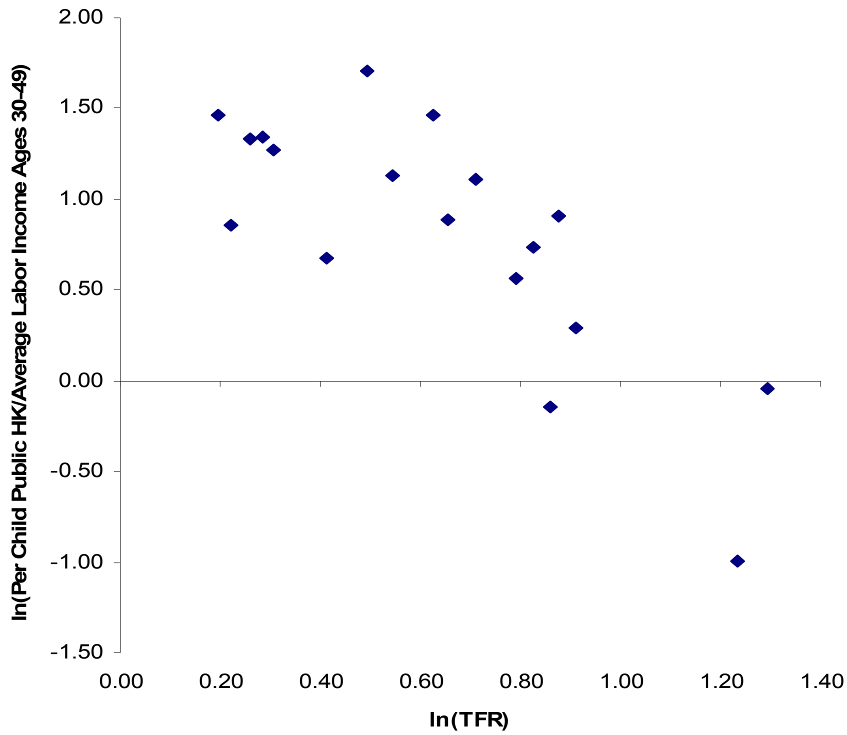


Figure 2. Private Per Child HK Spending vs. Fertility
ln(Per Child Private HK/Average Labor Income Ages 30-49)
2.00
1.50
1.00
0.50
0.00
-0.50
-1.00
-1.50
-2.00
0.00
0.20
0.40
0.60
0.80
1.00
1.20
1.40
ln(TFR)

Figure 3. Public Per Child HK Spending vs. Fertility



Figiure 4. Private and Public Spending as Share of Predicted HK Spending (based on regression for Figure 1)

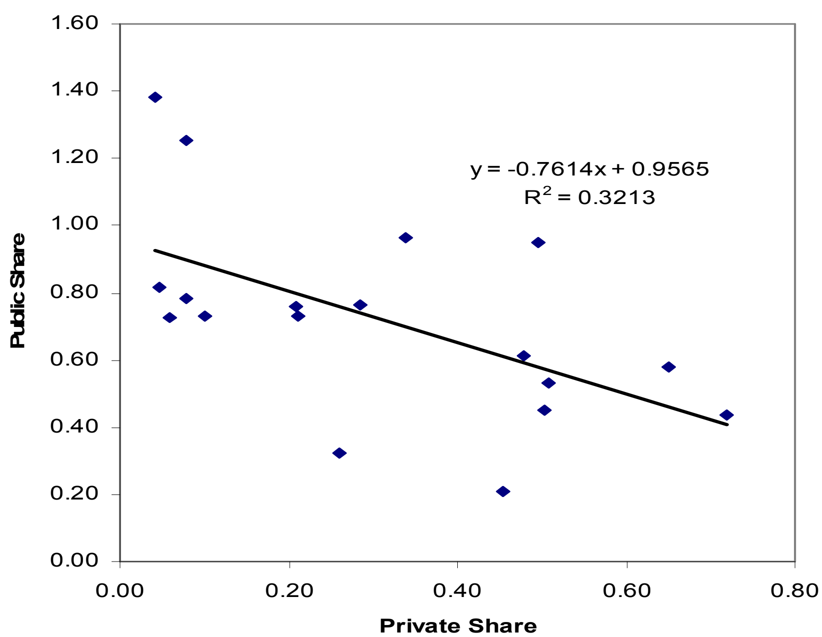


Figure 5. Total Expenditures Per Woman for All Children's HK vs. Fertility for 18 NTA countries (log scale)

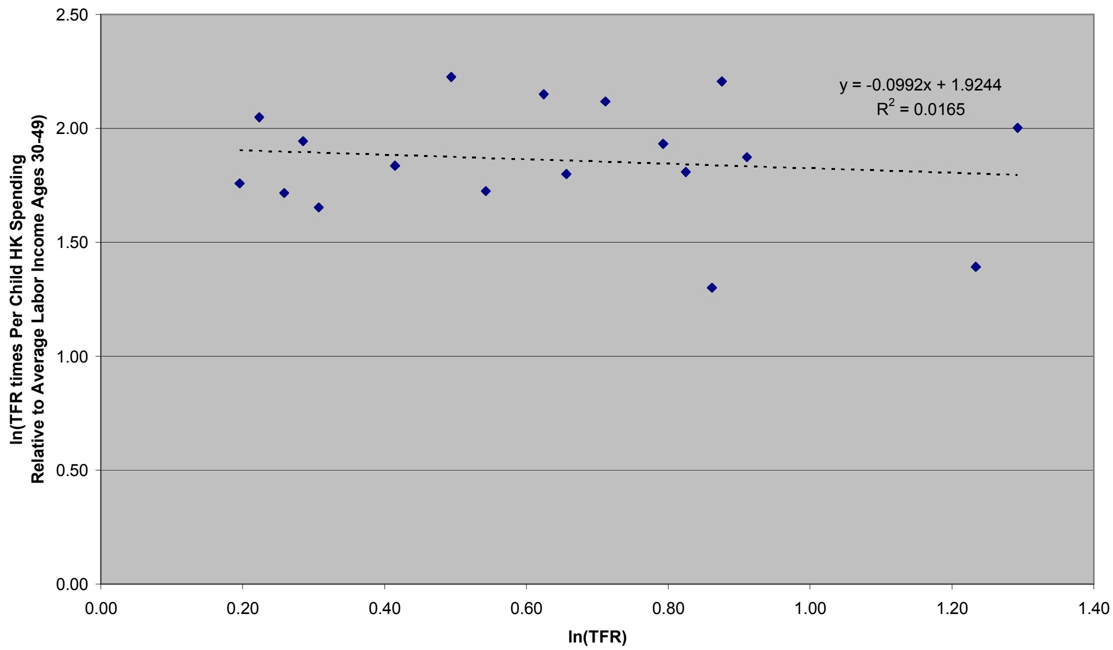


Figure 6A: The transformed budget constraint showing different quantity-quality choices.

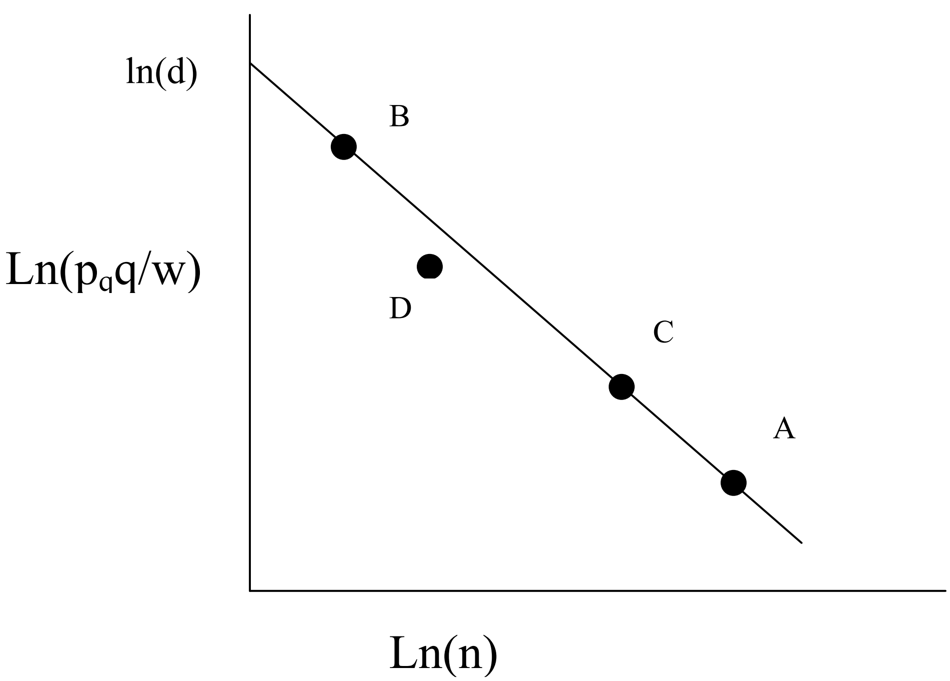


Figure 6. Macro Indicators: Baseline Results

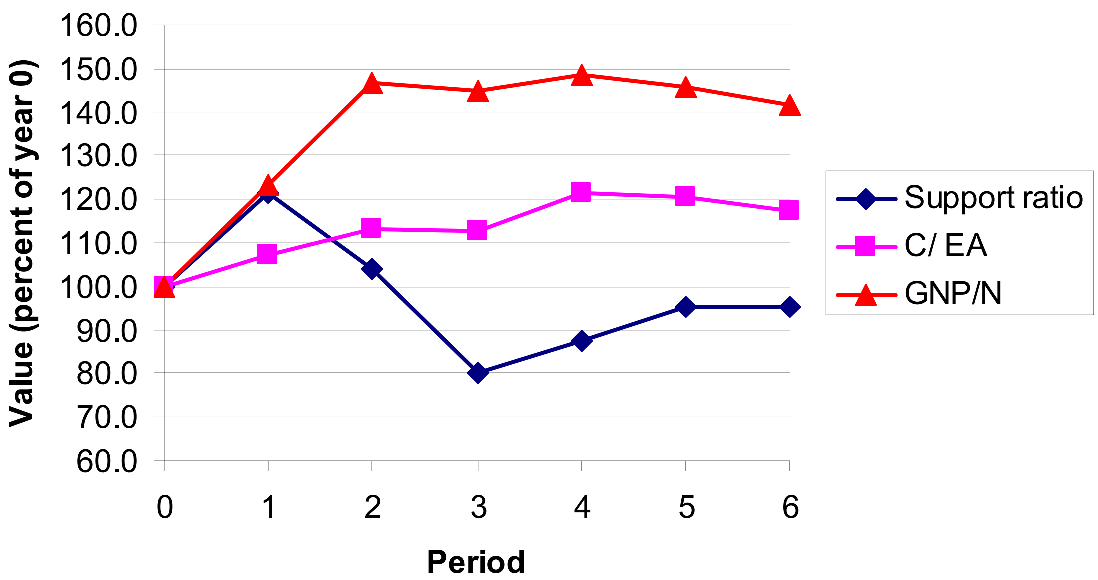
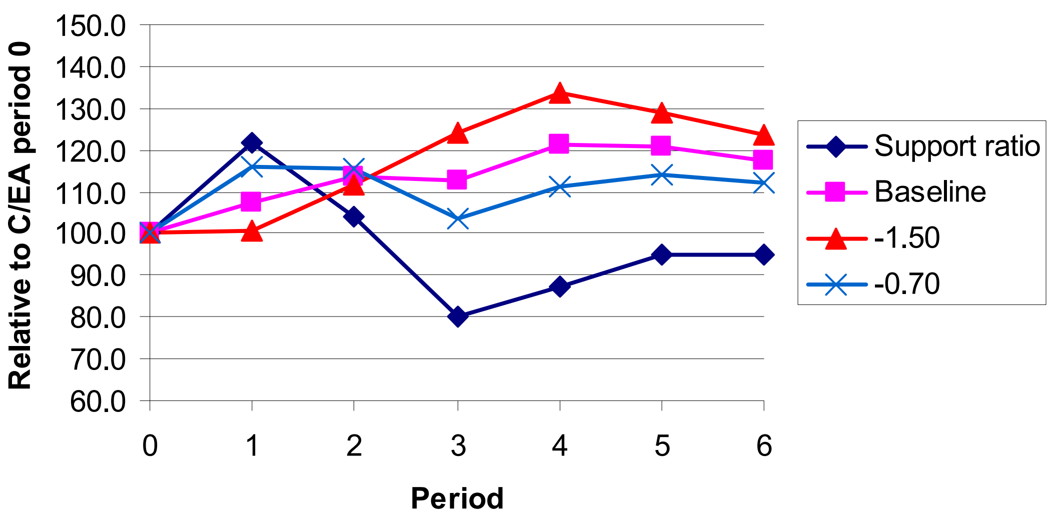


Figure 7. Consumption per equivalent adult, alternative elasticities of h wrt F



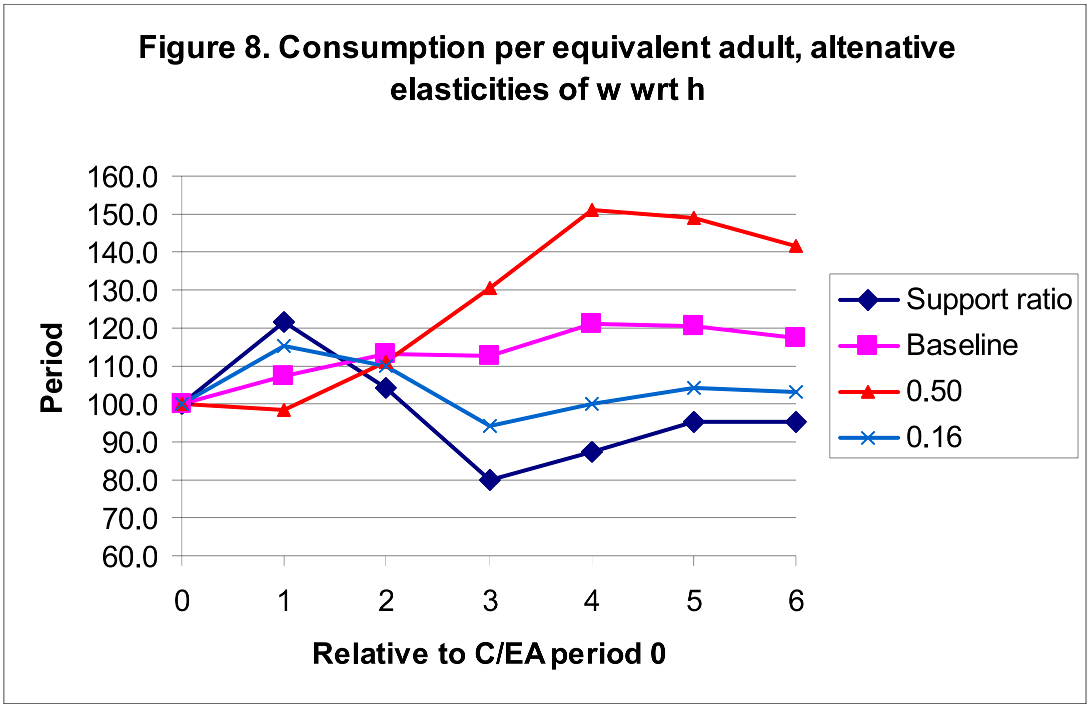


Figure 8. Consumption per equivalent adult, altenative elasticities of w wrt h
160.0
150.0
140.0
130.0
120.0
110.0
100.0
90.0
80.0
70.0
60.0
Period
0
1
2
3
4
5
6
Relative to C/EA period 0
Support ratio
Baseline
0.50
0.16

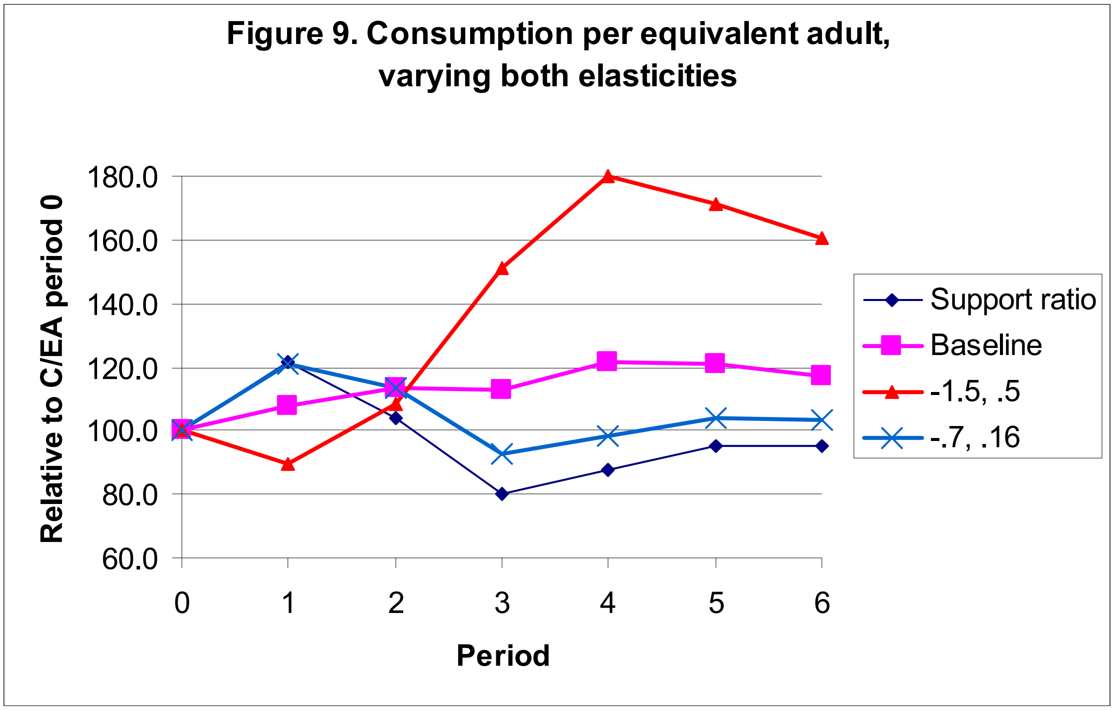


Figure 9. Consumption per equivalent adult,
varying both elasticities
Relative to C/EA period 0
180.0
160.0
140.0
120.0
100.0
80.0
60.0
0
1
2
3
4
5
6
Period
Support ratio
Baseline
-1.5, .5
-.7, .16

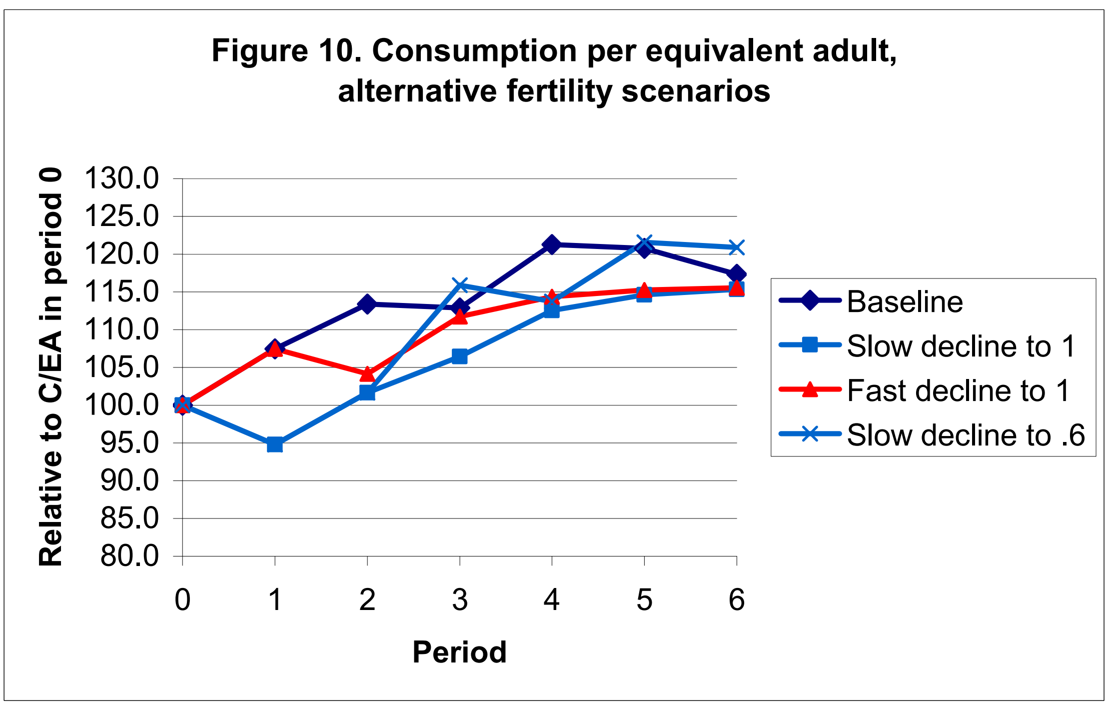


Figure 10. Consumption per equivalent adult,
alternative fertility scenarios
Relative to C/EA in period 0
130.0
125.0
120.0
115.0
110.0
105.0
100.0
95.0
90.0
85.0
80.0
0
1
2
3
4
5
6
Period
Baseline
Slow decline to 1
Fast decline to 1
Slow decline to .6

Table 1. Variables and their definitions.	
Nx(t); N(t)	Population age x, x = 0, 1, or 2; Total Population
F(t)	Fertility rate
s(t)	Adult survival: proportion of population N2(t-1) surviving to N3(t)
W(t)	Wage in year t
H(t)	Human capital per worker in year t
i(t)	Proportion of wage invested in each child's human capital in year t
T(t)	GNP (wage income only) in year t
C(t)	Total consumption in year t
c(t)	Consumption per equivalent adult in year t

Table 2. Parameter values and sources.		
	Value	**Source**
α	0.1	In data spending was 3.8 years worth of prime adult labor income; total years of prime age adult labor was 39.4. The mean TFR was approximately replacement (actually 1.9). Investment rate of 3.8/39.4 = approximately 0.1.
β	-1.1	Regression from NTA estimates. See text.
γ	1	Arbitrary (doesn't matter)
δ	0.33	Mankiw, Romer, and Weil
a_0	0.5	Estimated NTA consumption profile for developing countries.
a_2	1.0	Estimated NTA consumption profile for developing countries.

Table 3. Demographic Variables, Baseline Simulation

				Percent of population			
Period	NRR	Survival to old age	Growth rate	Children	Workers	Elderly	Support ratio
0	2.0	0.3	0.019	62.7	31.4	8.8	0.457
1	1.0	0.6	0.012	43.5	43.5	5.9	0.556
2	0.6	0.8	0.001	25.0	41.7	13.0	0.476
3	0.8	0.8	-0.008	25.5	31.9	33.3	0.366
4	1.0	0.8	-0.009	33.3	33.3	42.6	0.400
5	1.0	0.8	-0.002	35.7	35.7	33.3	0.435
6	1.0	0.8	0.000	35.7	35.7	28.6	0.435

Table 4. Human Capital Variables

Period		Human capital spending per child/Wage	Wage	Human capital spending per child	Average human capital of workers	Human capital spending/ GDP
0	Boom	0.047	0.263	0.012	0.017	0.093
1	Decline	0.100	0.234	0.023	0.012	0.100
2		0.175	0.290	0.051	0.023	0.105
3	Recovery	0.128	0.374	0.048	0.051	0.102
4		0.100	0.367	0.037	0.048	0.100
5		0.100	0.336	0.034	0.037	0.100
6		0.100	0.326	0.033	0.034	0.100

[1] Labor income at a given age is averaged across men and women, for all members of the population at that age, including those with zero income. Labor income includes wages and salaries, fringe benefits, self employment, and estimates for unpaid family labor which is very important in poor countries.